Multicell MISO Downlink Weighted Sum-Rate Maximization: A Distributed Approach

*P. C. Weeraddana
**M. Codreanu, **M. Latva-Aho, ***A. Ephremides
*KTH, Royal institute of Technology, Stockholm, Sweden
**CWC, University of Oulu, Finland
***University of Maryland, USA

2013.02.19

Motivation

- Centralized RA requires gathering problem data at a central location -> huge overhead

- Large-scale communication networks -> large-scale problems

- Distributed solution methods are indeed desirable
    - Many local subproblems -> small problems
    - Coordination between subproblems -> light protocol

Motivation

- WSRMax: a central component of many NW control and optimization methods, e.g.,
  - Cross-layer control policies
  - NUM for wireless networks
  - MaxWeight link scheduling for wireless networks
  - power and rate control policies for wireless networks
  - achievable rate regions in wireless networks

Challenges

- WSRMax problem is nonconvex, in fact NP-hard
  - At least a suboptimal solution is desirable
- Considering the most general wireless network (MANET) is indeed difficult
  - A particular case is infrastructure based wireless networks
  - Cellular networks
- Coordinating entities
  - MS-BS, MS-MS, BS-BS
- Coordination between subproblems -> light protocol

Our contribution

- **Distributed algorithm for WSRMax for MISO interfering BC channel; BS-BS coordination required**

- Algorithm is based on primal decomposition methods and subgradient methods

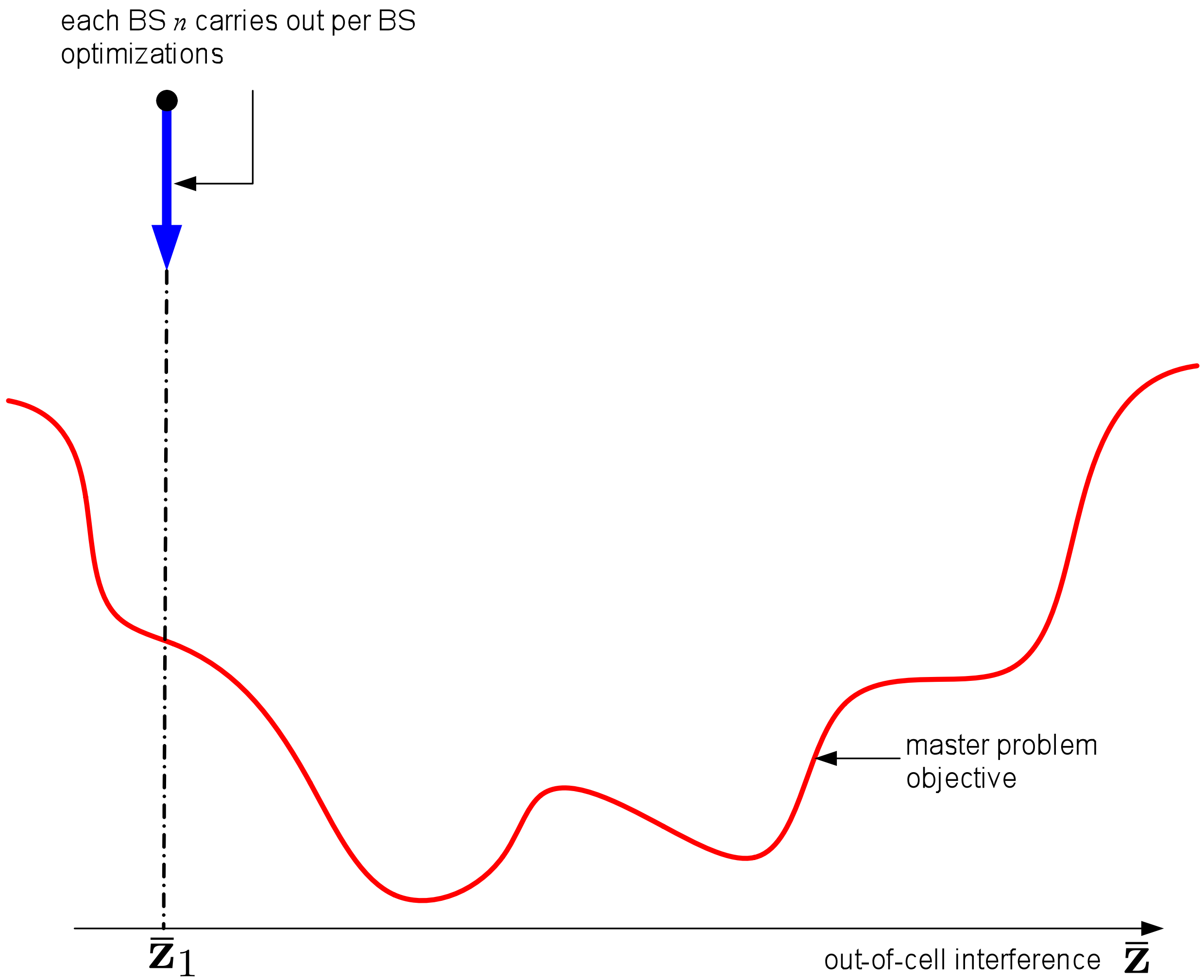
- Split the problem into subproblems and a master problem
  - local variables: Tx beamforming directions and power
  - global variables: out-of-cell interference power

- Subproblems asynchronous (one for each BS)
  - variables: Tx beamforming directions and power

- Master problem resolves out-of-cell interference (coupling)

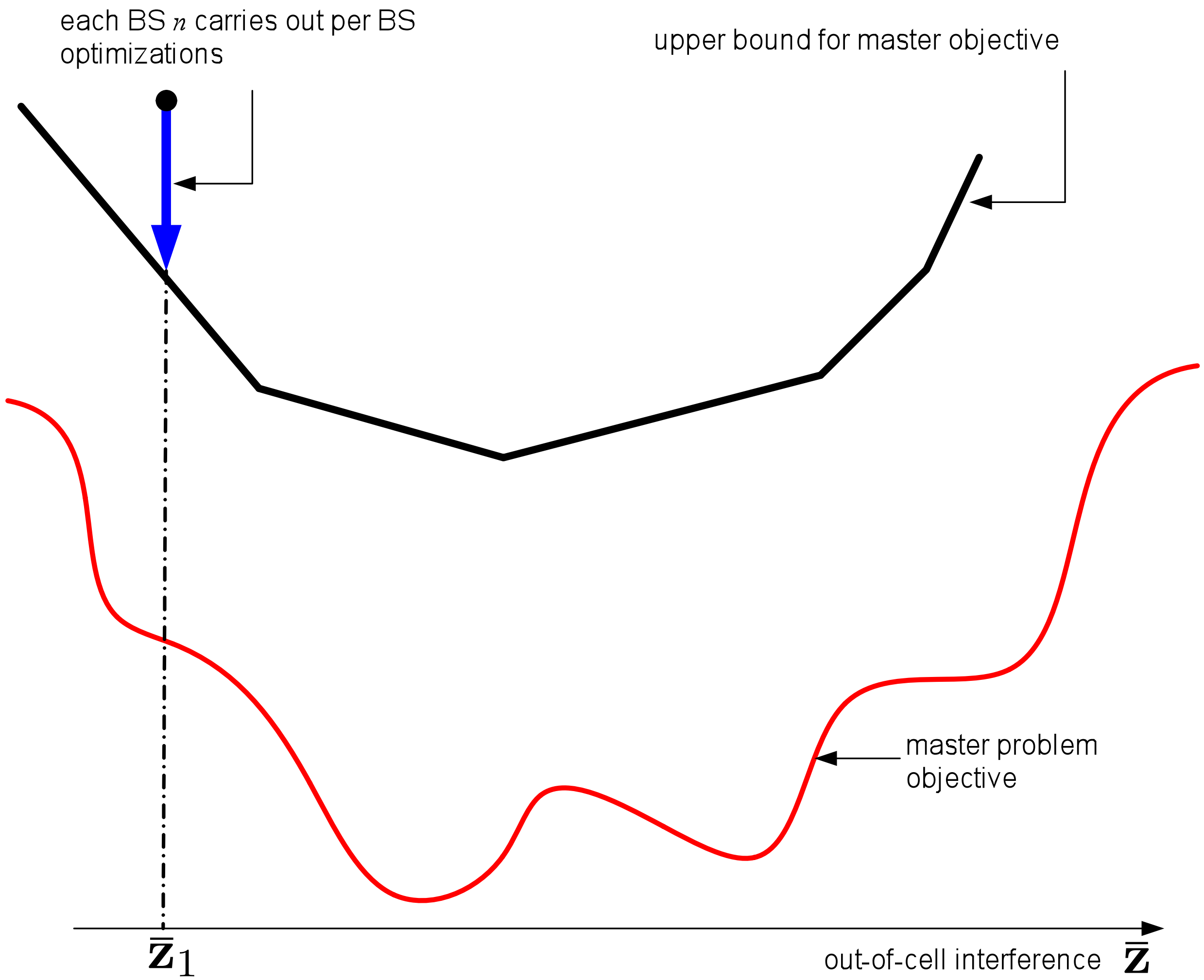
P. C. Weeraddana, M. Codreanu, M. Latva-aho, and A. Ephremides, IEEE Transactions on Signal Processing, February, 2013

Key Idea



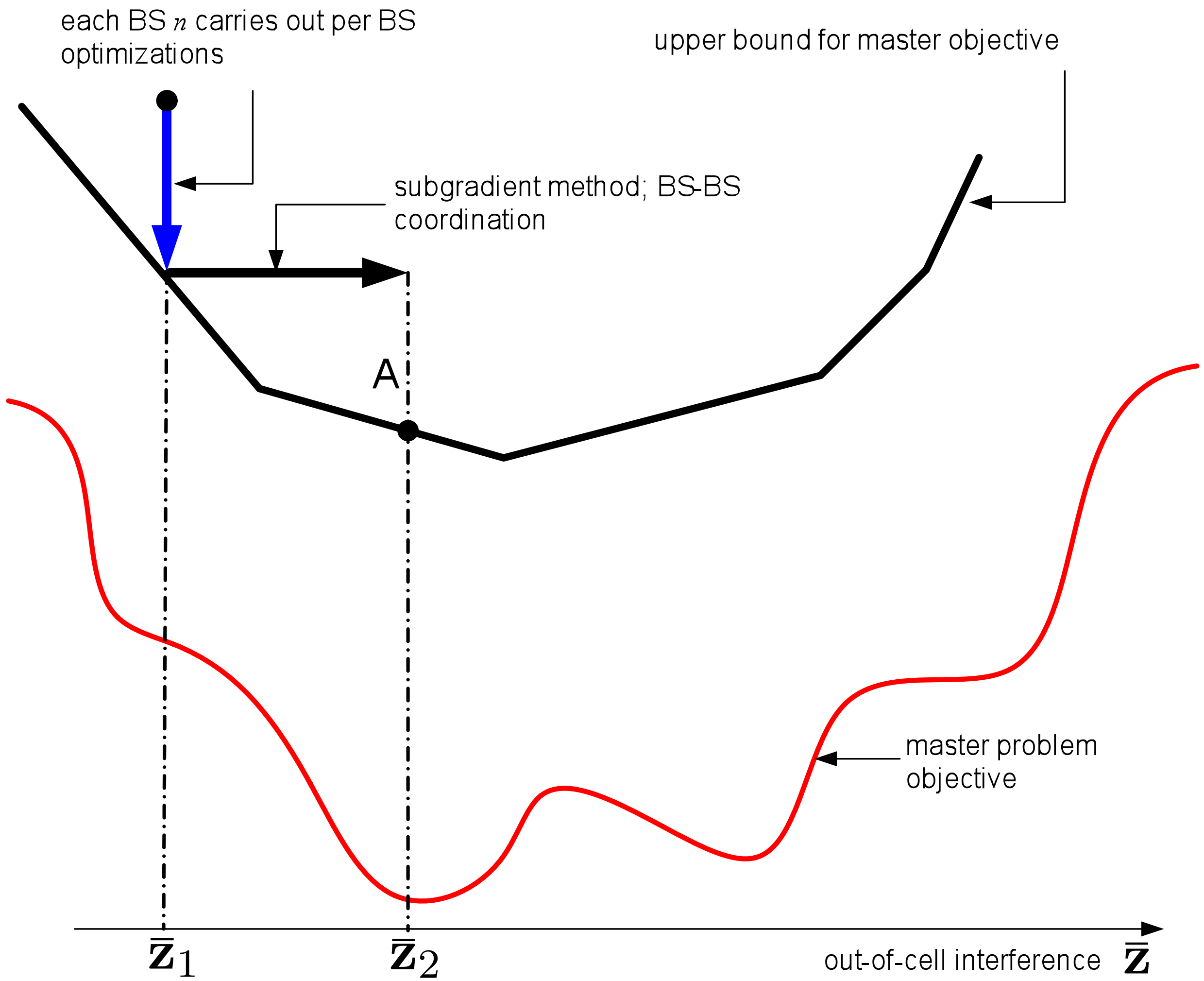
each BS n carries out per BS optimizations
master problem objective
out-of-cell interference $\bar{\mathbf{z}}$
$\bar{\mathbf{z}}_1$

Key Idea



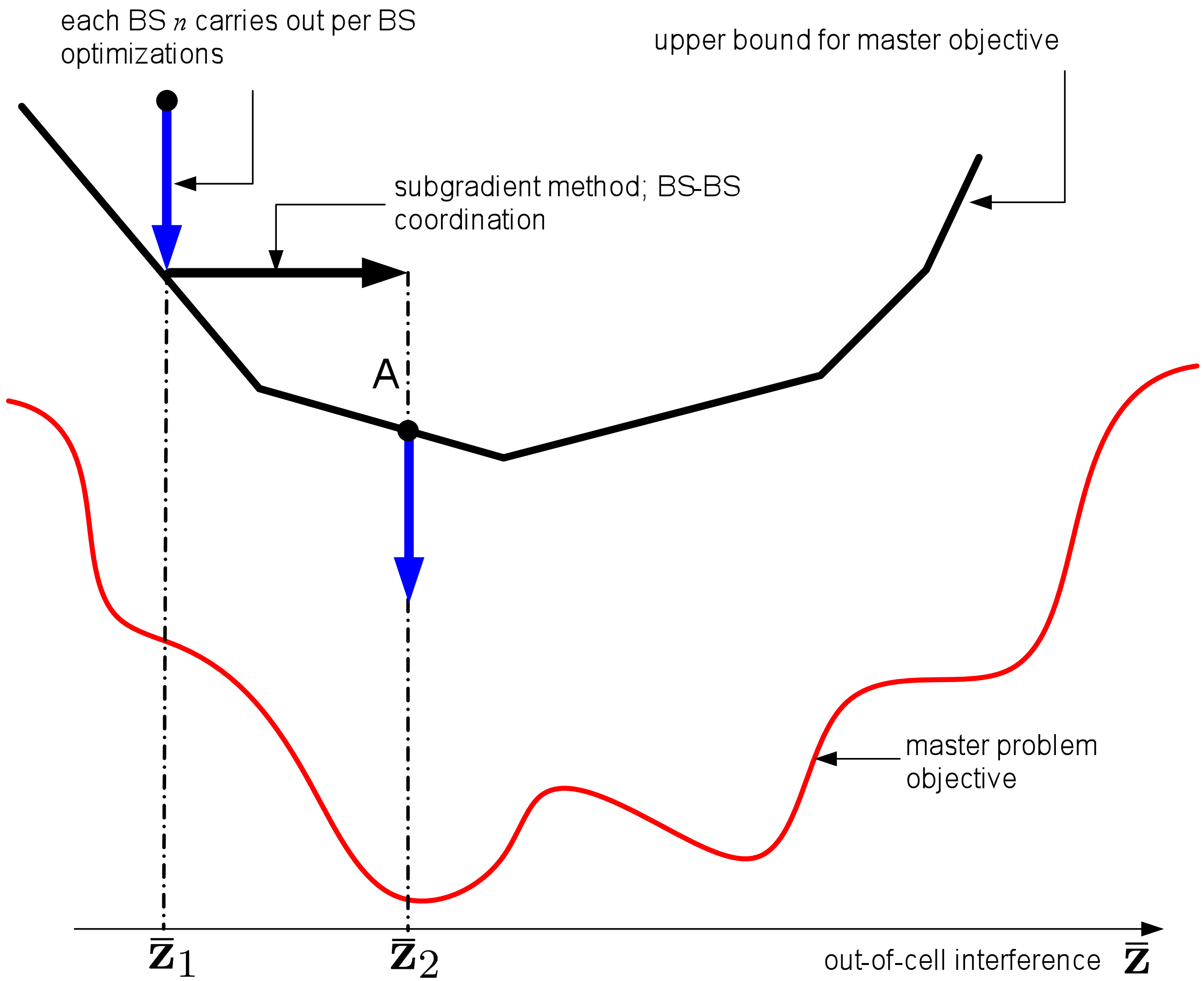
each BS n carries out per BS optimizations
upper bound for master objective
master problem objective
$\bar{\mathbf{z}}_1$
out-of-cell interference $\bar{\mathbf{z}}$

Key Idea



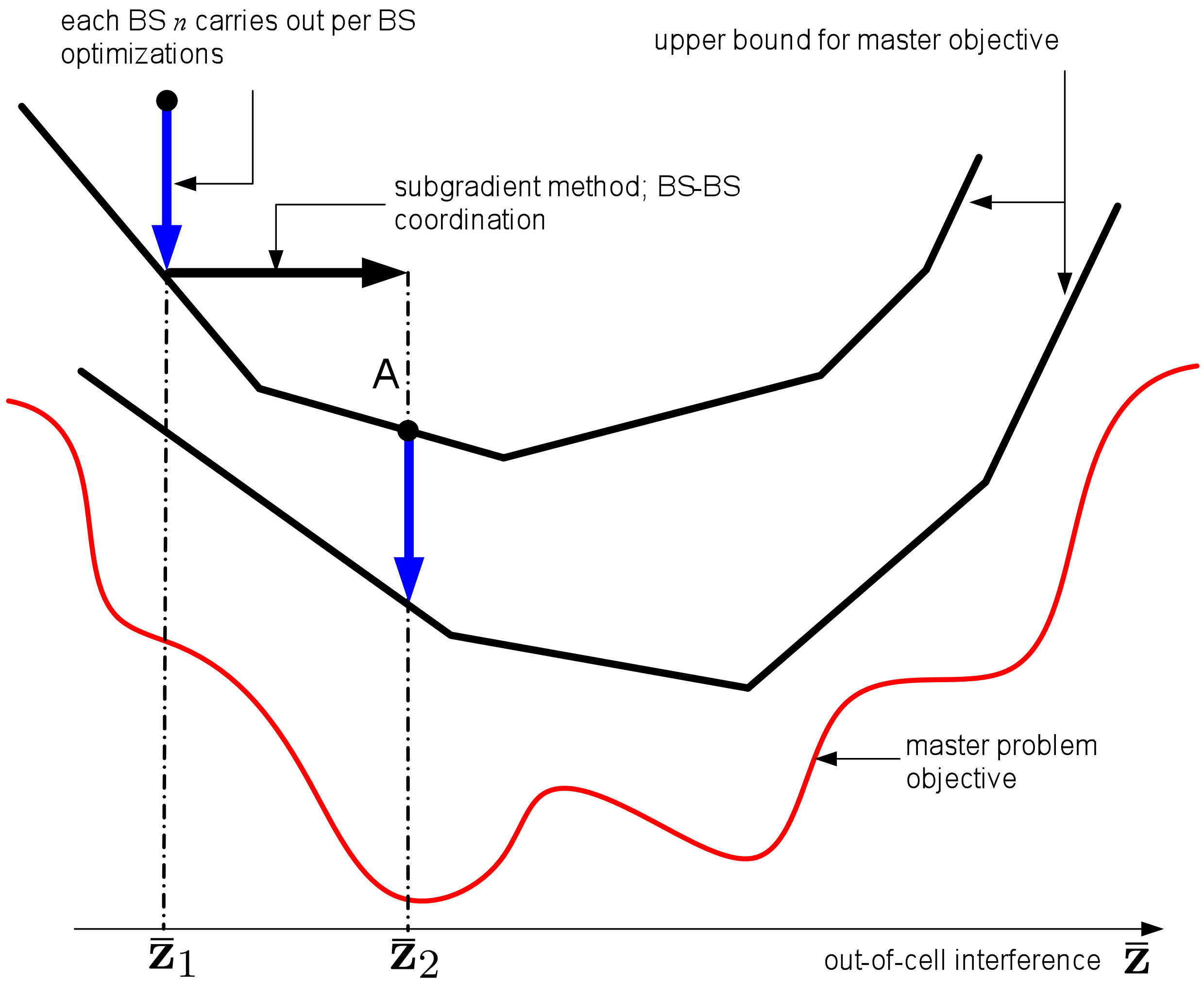
each BS n carries out per BS optimizations
upper bound for master objective
subgradient method; BS-BS coordination
A
master problem objective
$\bar{\mathbf{z}}_1$
$\bar{\mathbf{z}}_2$
out-of-cell interference $\bar{\mathbf{z}}$

Key Idea



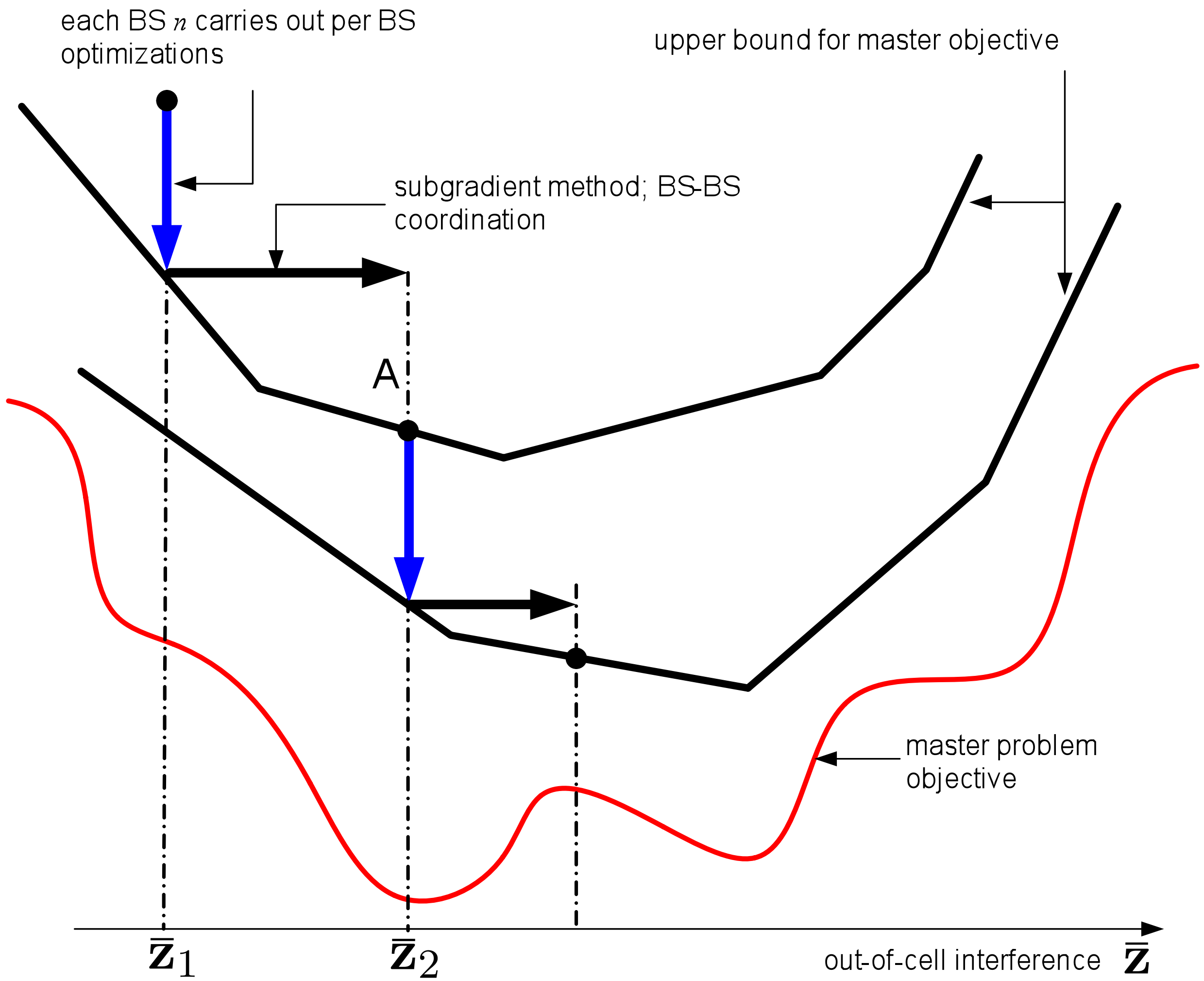
each BS n carries out per BS optimizations
upper bound for master objective
subgradient method; BS-BS coordination
A
master problem objective
$\bar{\mathbf{z}}_1$
$\bar{\mathbf{z}}_2$
out-of-cell interference $\bar{\mathbf{z}}$

Key Idea



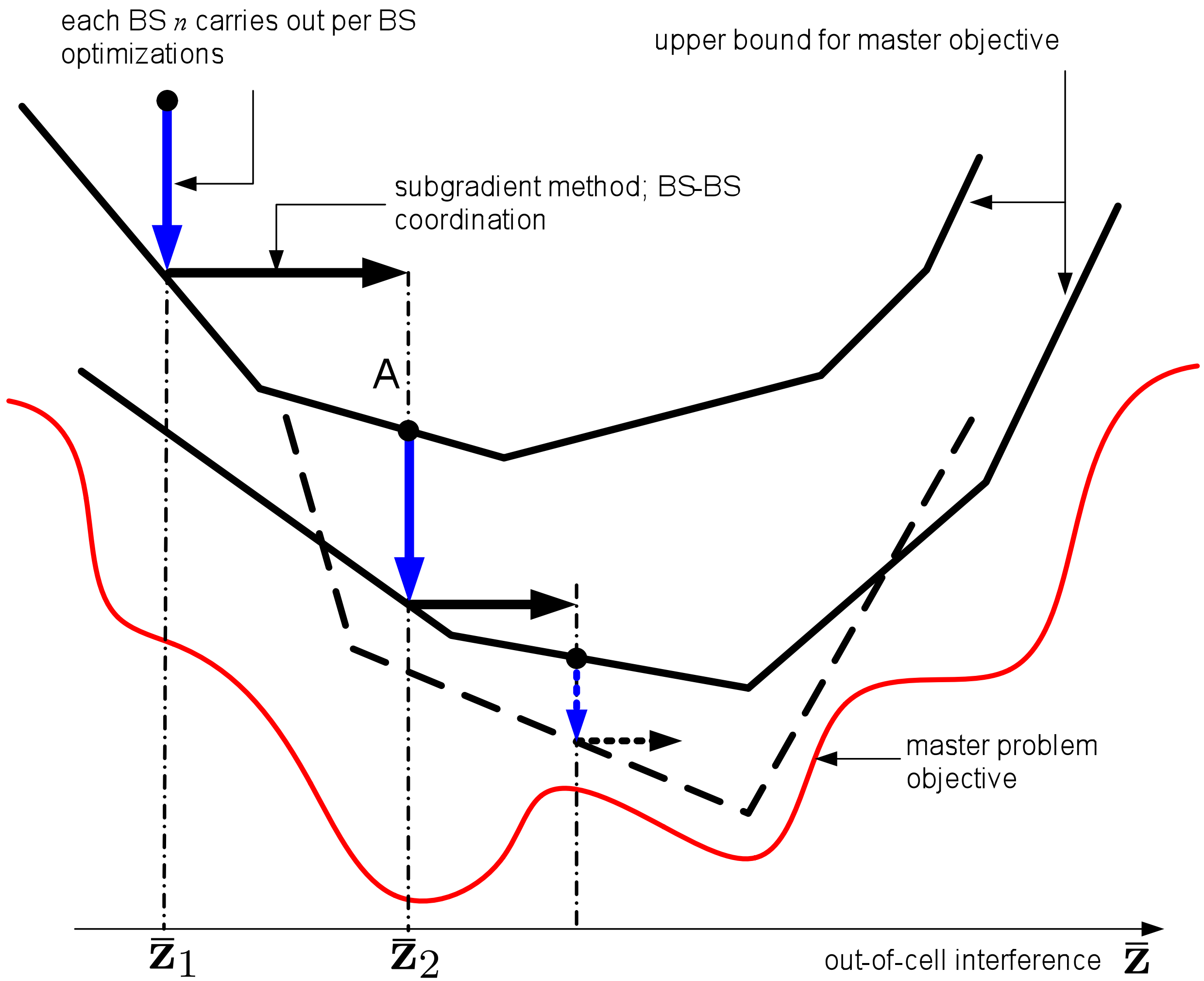
each BS n carries out per BS optimizations
upper bound for master objective
subgradient method; BS-BS coordination
A
master problem objective
$\bar{\mathbf{z}}_1$
$\bar{\mathbf{z}}_2$
out-of-cell interference $\bar{\mathbf{z}}$

Key Idea



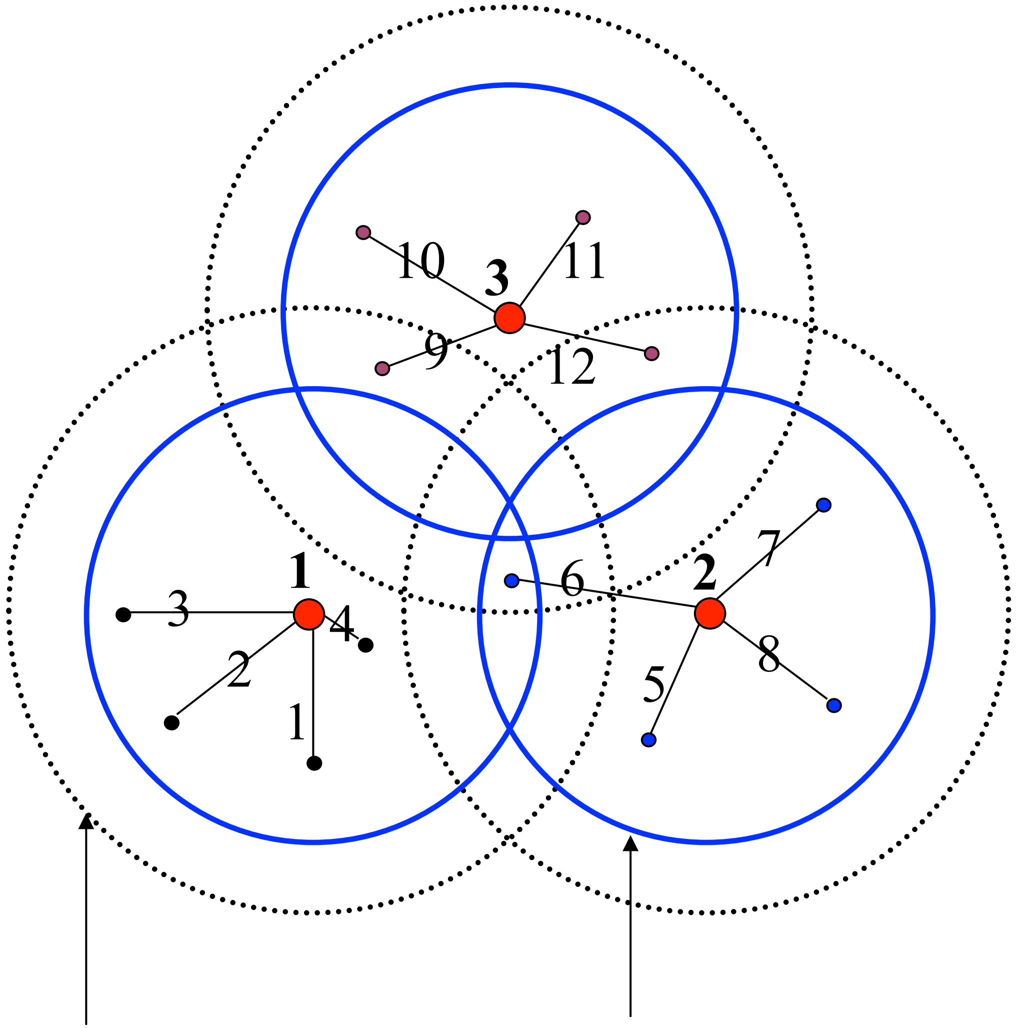
each BS n carries out per BS optimizations
upper bound for master objective
subgradient method; BS-BS coordination
A
master problem objective
$\bar{\mathbf{z}}_1$
$\bar{\mathbf{z}}_2$
out-of-cell interference $\bar{\mathbf{z}}$

Key Idea



each BS n carries out per BS optimizations
upper bound for master objective
subgradient method; BS-BS coordination
A
master problem objective
$\bar{\mathbf{z}}_1$
$\bar{\mathbf{z}}_2$
out-of-cell interference $\bar{\mathbf{z}}$

System model



N : number of BSs

$\mathcal{N}$: set of BSs

L : number of data streams

$\mathcal{L}$: set of data streams

$\mathcal{L}(n)$: set of data streams of BS n

T : number of BS antennas

$rec(l)$: receiver node of d.s. l

$tran(l)$: transmitter node of d.s. l

System model

signal vector transmitted by BS n

$$\mathbf{x}_n = \sum_{l \in \mathcal{L}(n)} \sqrt{p_l}\, d_l \mathbf{v}_l$$

p_l : power

d_l : information symbol; $\mathrm{E}|d_l|^2 = 1$, $\mathrm{E}\{d_l d_j^*\} = 0$

$\mathbf{v}_l$: beamforming vector; $\|\mathbf{v}_l\|_2 = 1$

System model

signal received at $rec(l)$

$$y_l = \mathbf{h}_{ll}^{\mathrm{H}} \sqrt{p_l}\, d_l \mathbf{v}_l + \overbrace{\sum_{j \in \mathcal{L}(tran(l)), j \neq l} \mathbf{h}_{jl}^{\mathrm{H}} \sqrt{p_j}\, d_j \mathbf{v}_j}^{\text{intra-cell interference}}$$

$$+ \underbrace{\sum_{i \in \mathcal{N} \setminus \{tran(l)\}} \sum_{j \in \mathcal{L}(i)} \mathbf{h}_{jl}^{\mathrm{H}} \sqrt{p_j}\, d_j \mathbf{v}_j}_{\text{out-of-cell interference}} + z_l$$

$\mathbf{h}_{jl}^{\mathrm{H}}$: channel; $tran(j)$ to $rec(l)$

z_l : cir. symm. complex Gaussian noise; variance σ_l^2

System model

received SINR of $rec(l)$

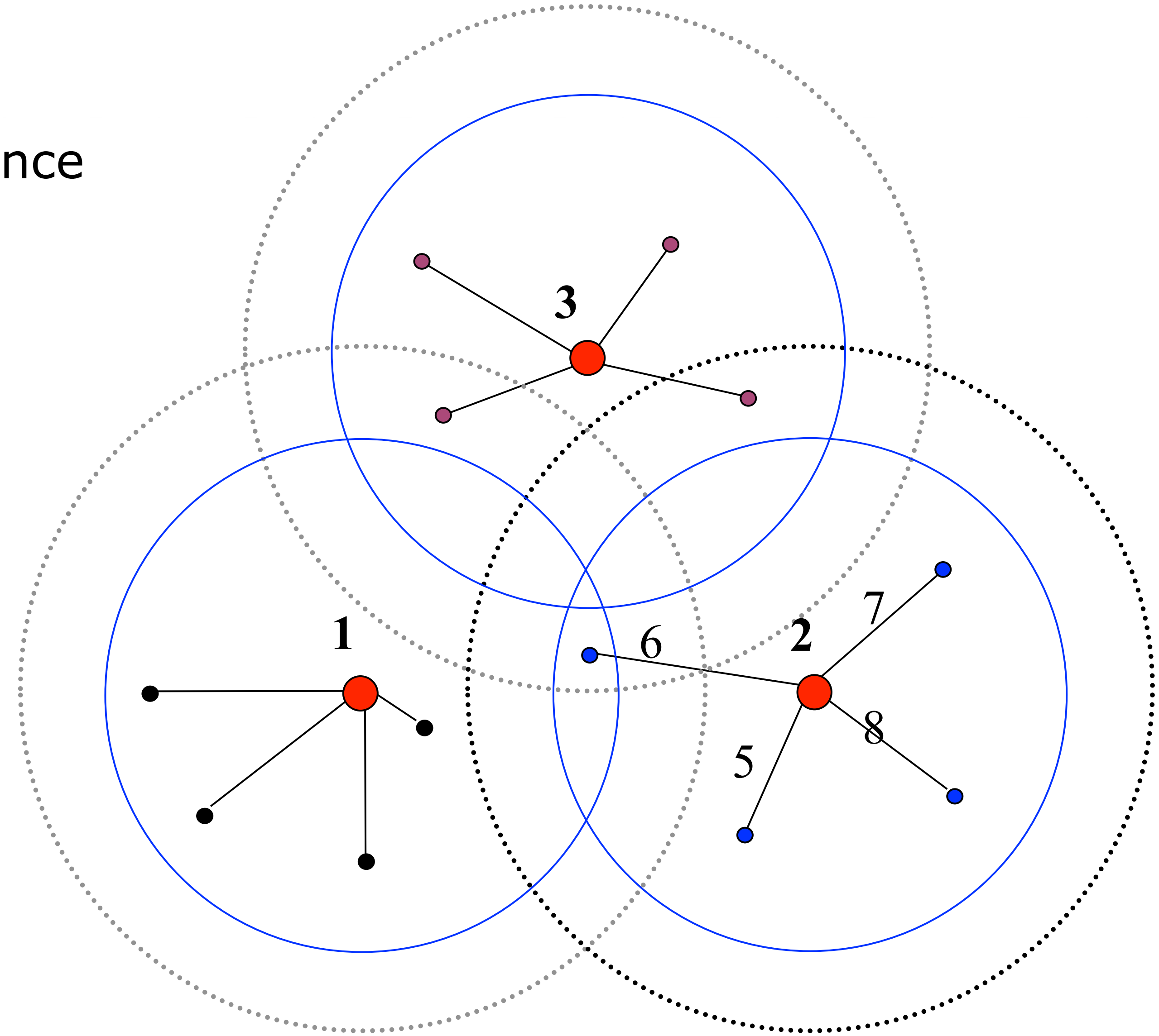
$$\gamma_l = \frac{p_l |\mathbf{h}_{ll}^{\mathrm{H}} \mathbf{v}_l|^2}{\sigma_l^2 + \underbrace{\sum_{j \in \mathcal{L}(tran(l)), j \neq l} p_j |\mathbf{h}_{ll}^{\mathrm{H}} \mathbf{v}_j|^2}_{\text{intra-cell interference}} + \underbrace{\sum_{i \in \mathcal{N} \setminus \{tran(l)\}} z_{il}}_{\text{out-of-cell interference}}}$$

$z_{il} = \sum_{j \in \mathcal{L}(i)} p_j |\mathbf{h}_{jl}^{\mathrm{H}} \mathbf{v}_j|^2$: out-of-cell interference power; i th BS to $rec(l)$

System model

out-of-cell interference

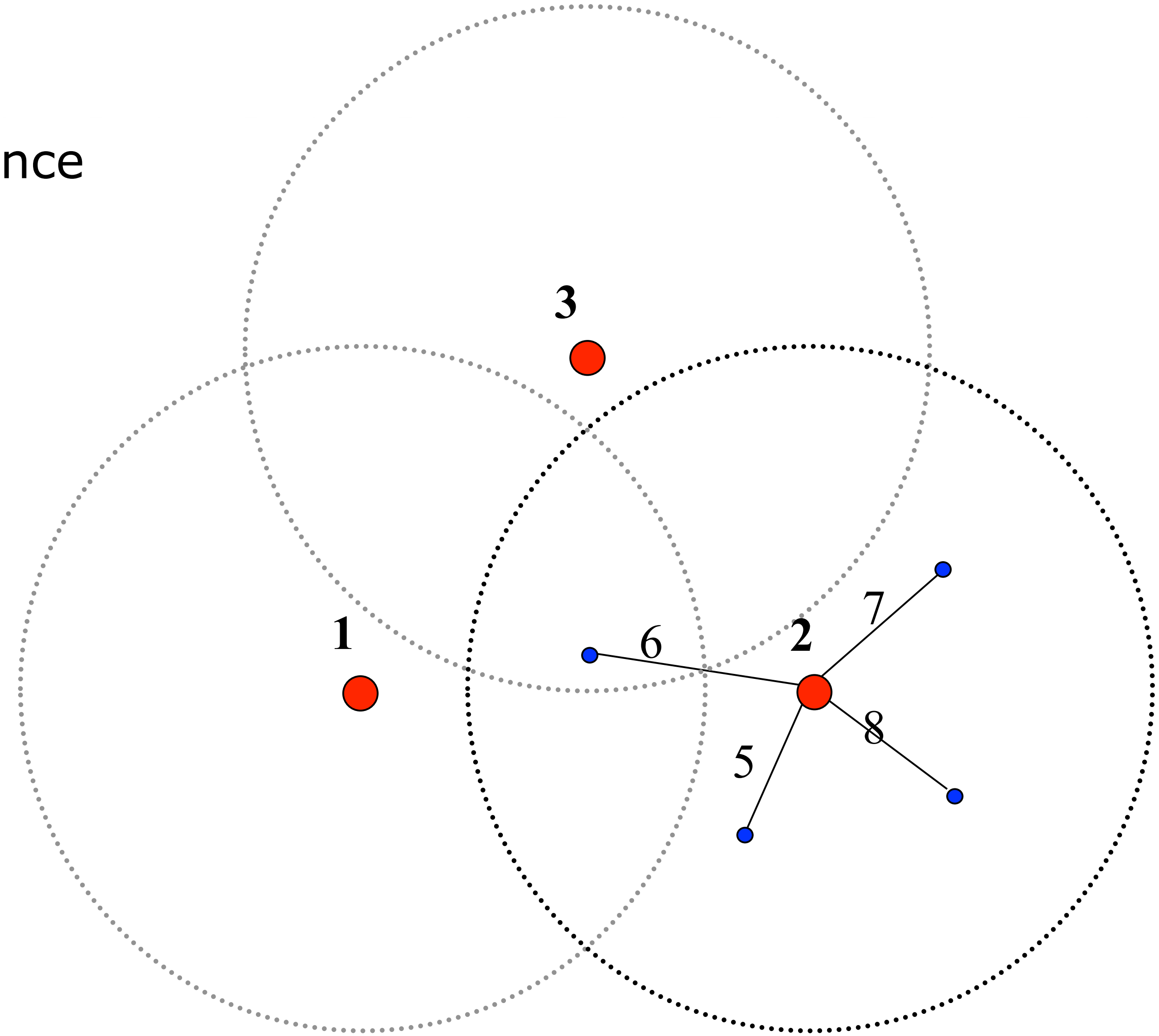
power, e.g.,



System model

out-of-cell interference

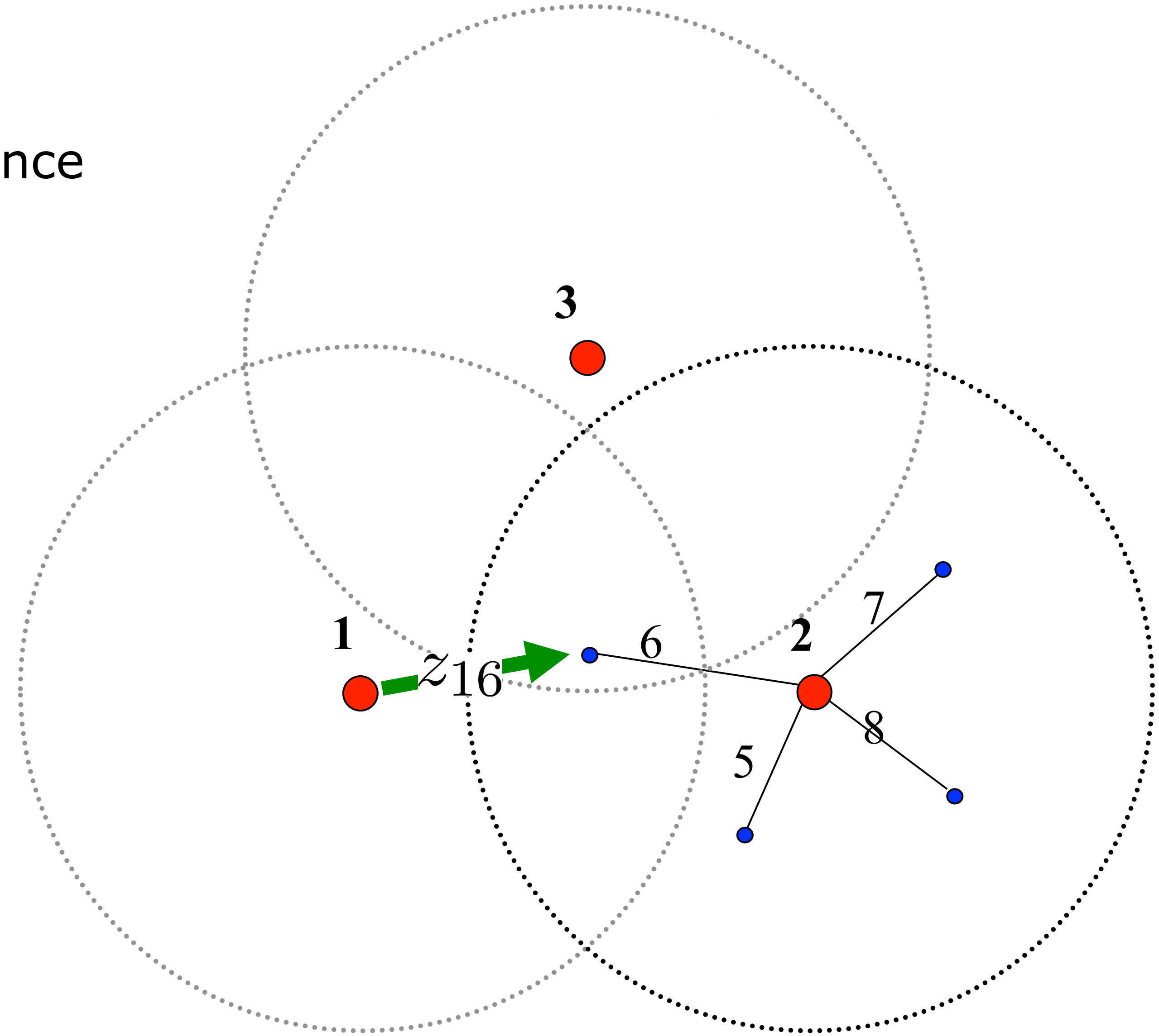
power, e.g.,



System model

out-of-cell interference

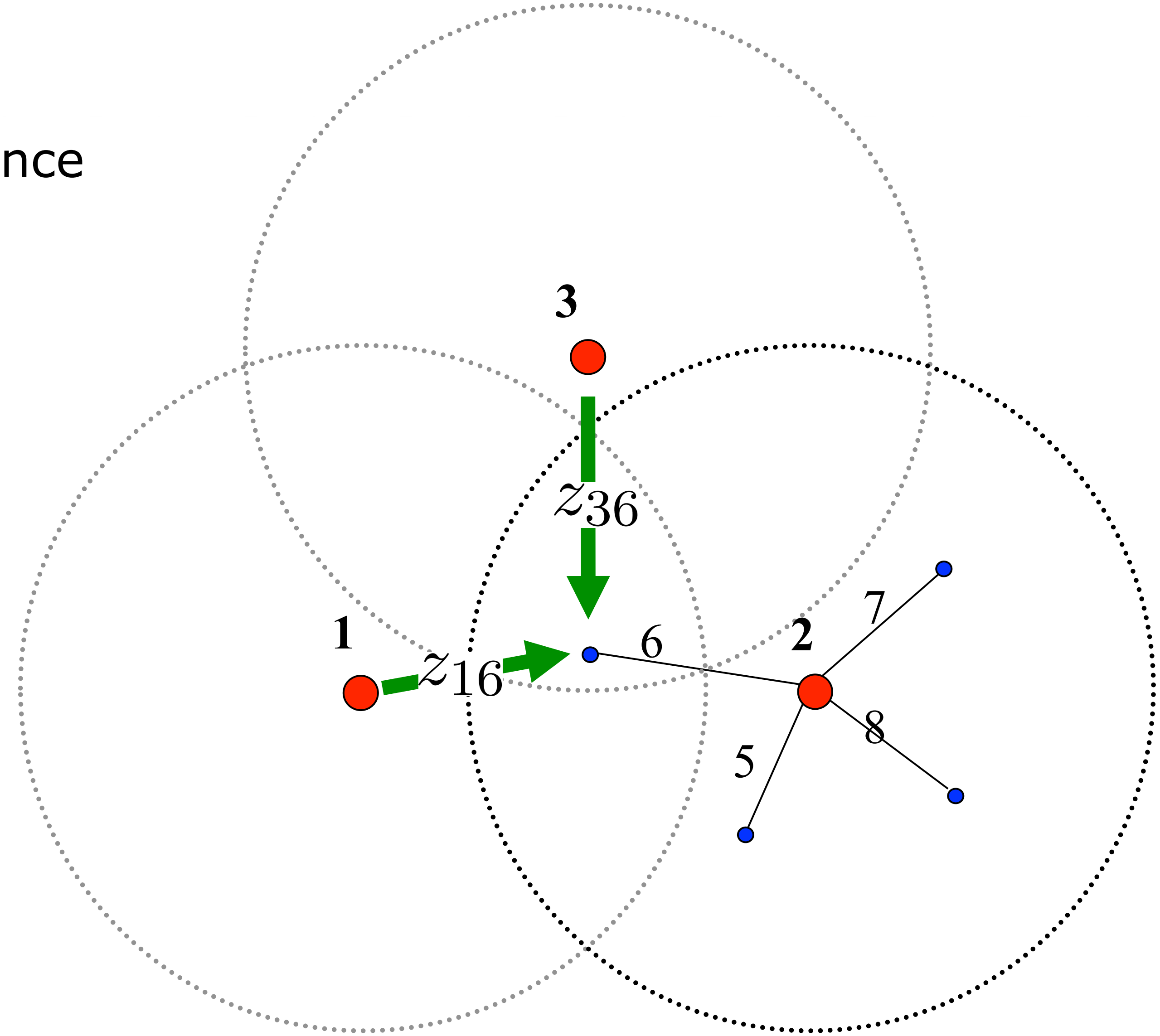
power, e.g.,



System model

out-of-cell interference

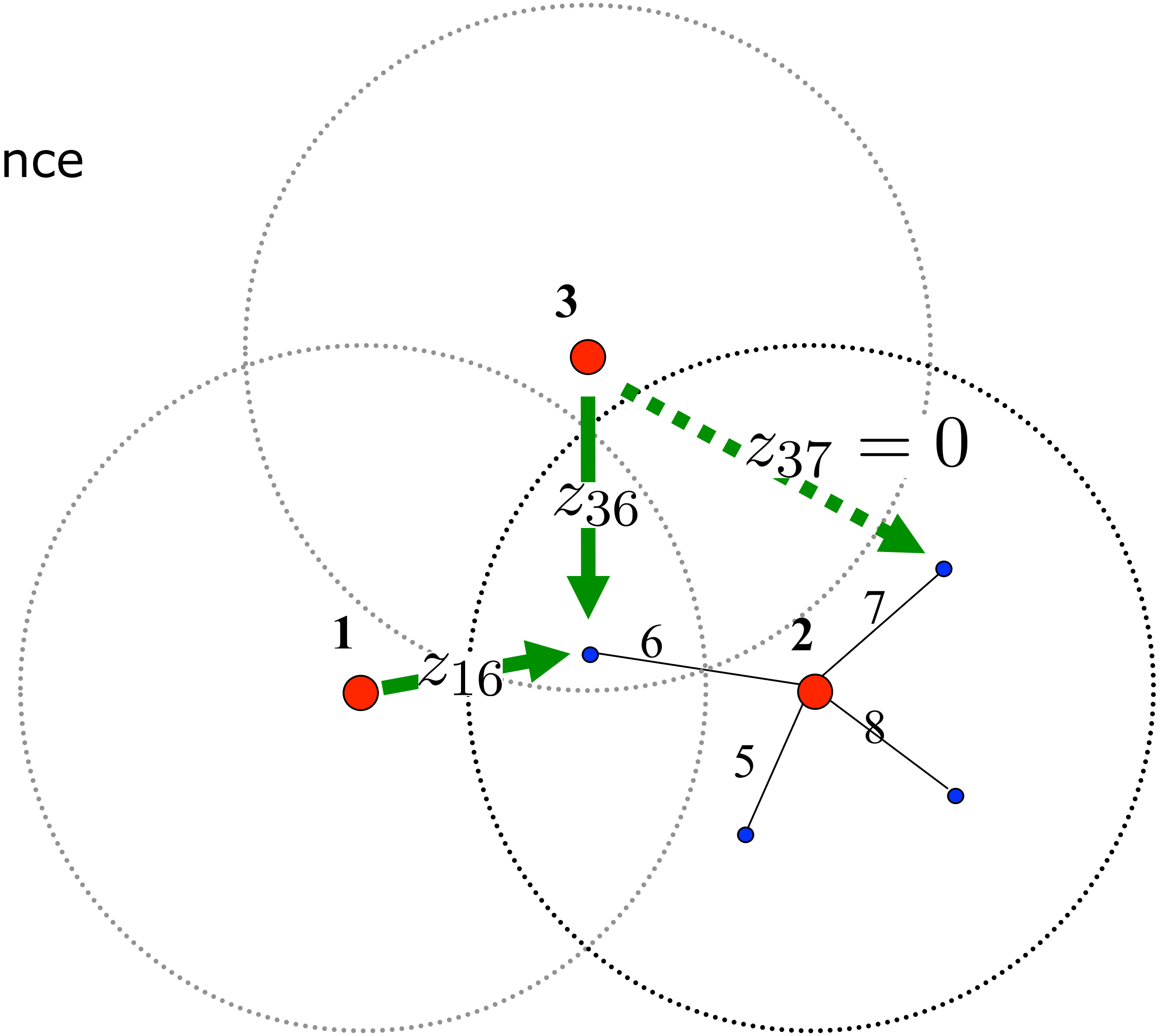
power, e.g.,



System model

out-of-cell interference

power, e.g.,



System model

received SINR of $rec(l)$

$$\gamma_l = \frac{p_l |\mathbf{h}_{ll}^{\mathrm{H}} \mathbf{v}_l|^2}{\sigma_l^2 + \underbrace{\sum_{j \in \mathcal{L}(tran(l)), j \neq l} p_j |\mathbf{h}_{ll}^{\mathrm{H}} \mathbf{v}_j|^2}_{\text{intra-cell interference}} + \underbrace{\sum_{i \in \mathcal{N} \backslash \{tran(l)\}} z_{il}}_{\text{out-of-cell interference}}}$$

$z_{il} = \sum_{j \in \mathcal{L}(i)} p_j |\mathbf{h}_{jl}^{\mathrm{H}} \mathbf{v}_j|^2$: out-of-cell interference power; i th BS to $rec(l)$

System model

received SINR of $rec(l)$

$$\gamma_l = \frac{p_l |\mathbf{h}_{ll}^{\mathrm{H}} \mathbf{v}_l|^2}{\sigma_l^2 + \underbrace{\sum_{j \in \mathcal{L}(tran(l)), j \neq l} p_j |\mathbf{h}_{ll}^{\mathrm{H}} \mathbf{v}_j|^2}_{\text{intra-cell interference}} + \underbrace{\sum_{i \in \mathcal{N}_{\mathrm{int}}(l)} z_{il}}_{\text{out-of-cell interference}}}$$

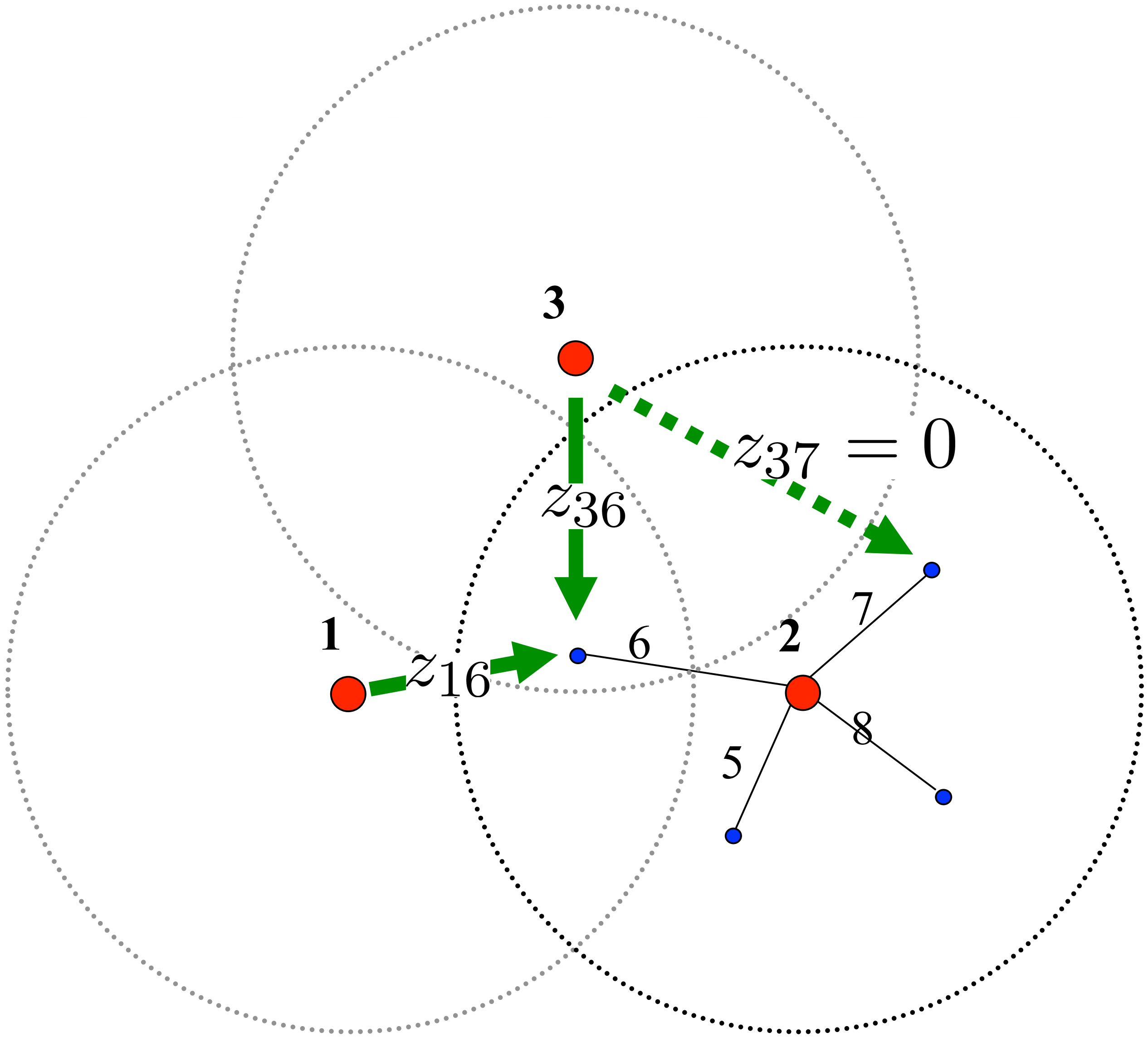
z_{il} : out-of-cell interference power (complicating variables)

$\mathcal{N}_{\mathrm{int}}(l)$: set of out-of-cell interfering BSs that interferes $rec(l)$

System model

e.g.,

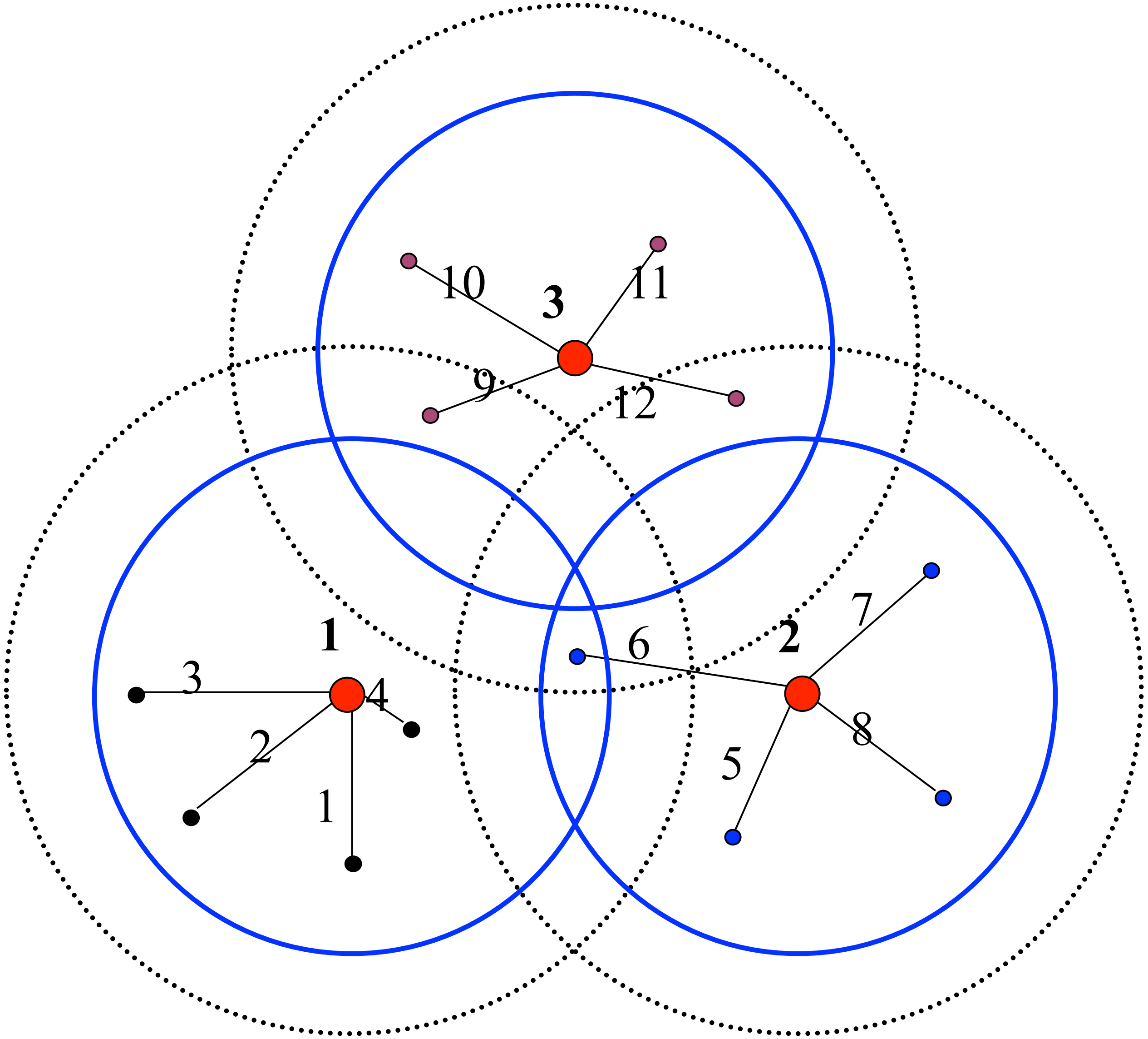
$$\mathcal{N}_{\text{int}}(6) = \{1, 3\}$$
$$\mathcal{N}_{\text{int}}(5) = \emptyset$$
$$\mathcal{N}_{\text{int}}(7) = \emptyset$$
$$\mathcal{N}_{\text{int}}(8) = \emptyset$$



System model

e.g.,

$$\mathcal{L}_{\text{int}} = \{6, 9, 12\}$$

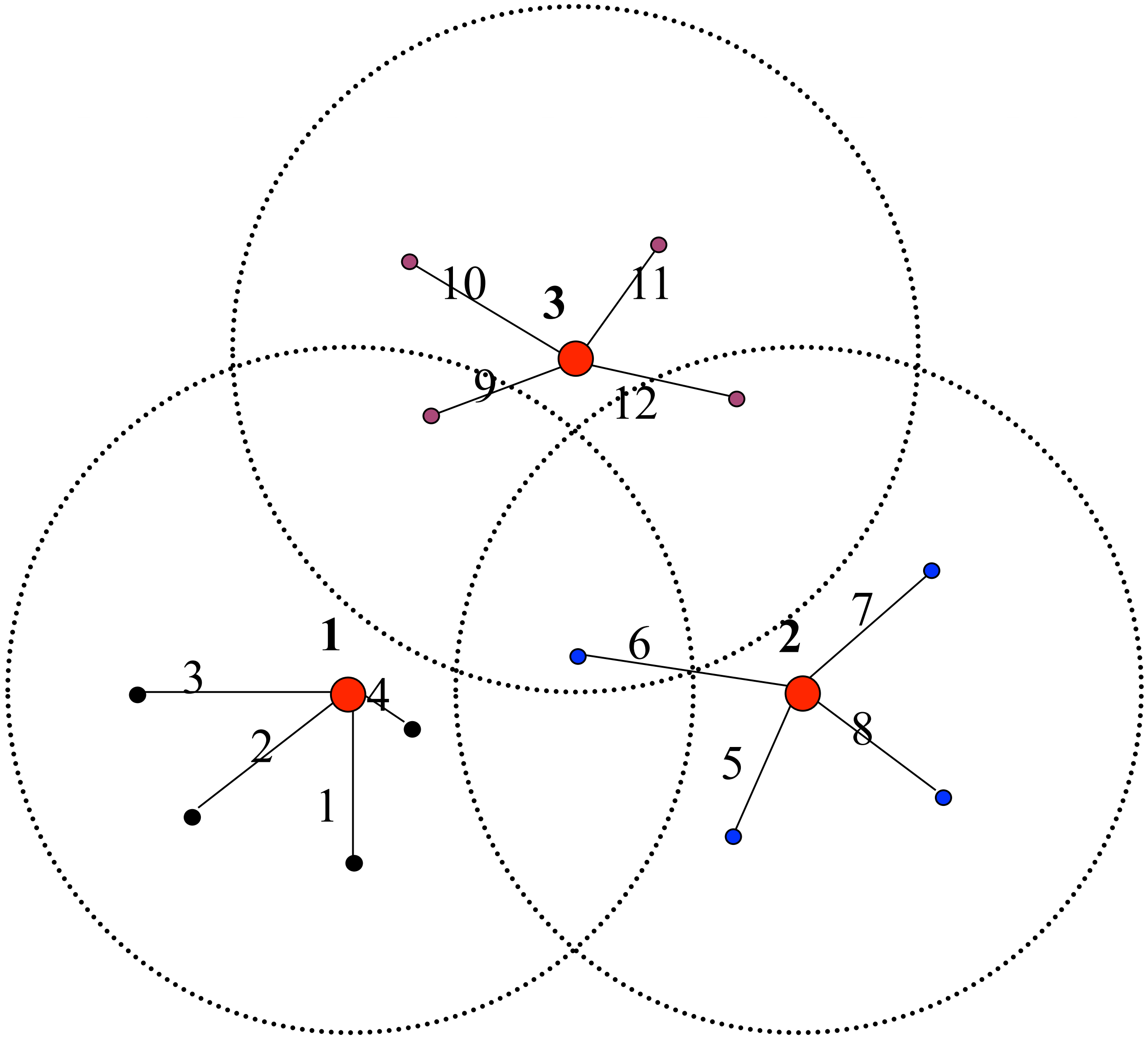


$\mathcal{L}_{\text{int}}$: set of d.s. that are subject to out-of-cell interference

System model

e.g.,

$$\mathcal{L}_{\text{int}} = \{6, 9, 12\}$$

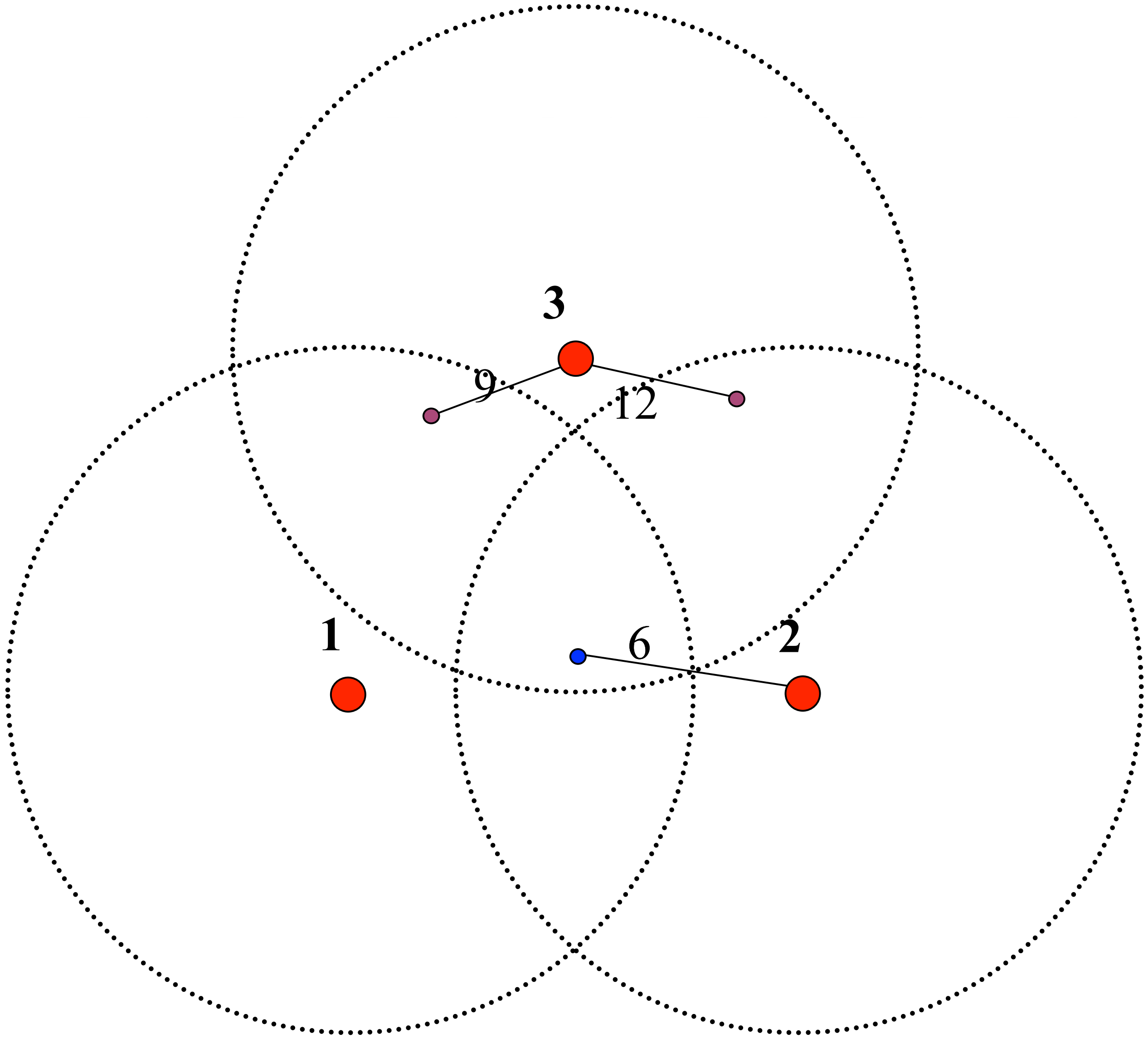


$\mathcal{L}_{\text{int}}$: set of d.s. that are subject to out-of-cell interference

System model

e.g.,

$$\mathcal{L}_{\text{int}} = \{6, 9, 12\}$$
$$\mathcal{L}_{\text{int}}(1) = \{6, 9\}$$
$$\mathcal{L}_{\text{int}}(2) = \{12\}$$
$$\mathcal{L}_{\text{int}}(3) = \{6\}$$



$\mathcal{L}_{\text{int}}$: set of d.s. that are subject to out-of-cell interference

Problem formulation

$$\text{minimize} \; -\sum_{n\in\mathcal{N}} \sum_{l\in\mathcal{L}(n)} \beta_l \log\left(1+\frac{p_l|\mathbf{h}_{ll}^{\mathrm{H}}\mathbf{v}_l|^2}{\sigma_l^2+\sum\limits_{j\in\mathcal{L}(tran(l)),j\neq l} p_j|\mathbf{h}_{ll}^{\mathrm{H}}\mathbf{v}_j|^2+\sum\limits_{i\in\mathcal{N}_{\text{int}}(l)} z_{il}}\right)$$

$$\text{subject to} \quad z_{il}=\sum_{j\in\mathcal{L}(i)} p_j|\mathbf{h}_{jl}^{\mathrm{H}}\mathbf{v}_j|^2,\; l\in\mathcal{L}_{\text{int}},\; i\in\mathcal{N}_{\text{int}}(l)$$

$$\sum_{l\in\mathcal{L}(n)} p_l||\mathbf{v}_l||_2^2 \leq p_n^{\max},\; n\in\mathcal{N}$$

$$||\mathbf{v}_l||_2=1, \quad p_l\geq 0,\; l\in\mathcal{L}\;,$$

variables: $\{p_l,\mathbf{v}_l\}_{l\in\mathcal{L}}$ and $\{z_{il}\}_{l\in\mathcal{L}_{\text{int}},i\in\mathcal{N}_{\text{int}}(l)}$

Primal decomposition

subproblems (for all $n \in \mathcal{N}$) :

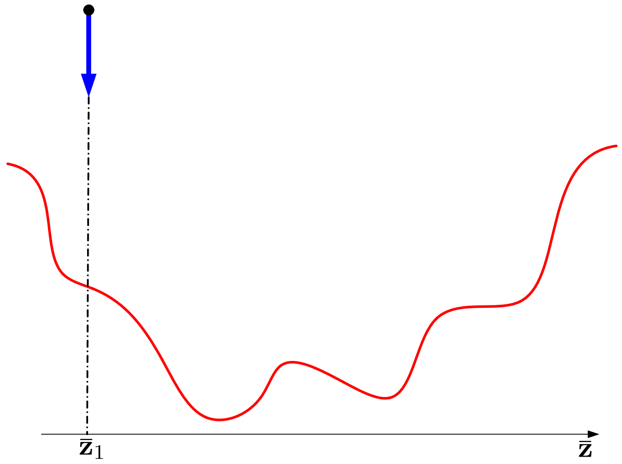
$$\begin{array}{ll} \text{minimize} & -\sum_{l\in\mathcal{L}(n)} \beta_l \log\left(1 + \dfrac{p_l|\mathbf{h}_{ll}^{\mathrm{H}}\mathbf{v}_l|^2}{\sigma_l^2 + \sum\limits_{j\in\mathcal{L}(n),j\neq l} p_j|\mathbf{h}_{ll}^{\mathrm{H}}\mathbf{v}_j|^2 + \sum\limits_{i\in\mathcal{N}_{\mathrm{int}}(l)} z_{il}}\right) \\ \text{subject to} & z_{il} \geq \sum_{j\in\mathcal{L}(i)} p_j|\mathbf{h}_{jl}^{\mathrm{H}}\mathbf{v}_j|^2, \;\; l \in \mathcal{L}_{\mathrm{int}}(n) \\ & \sum_{l\in\mathcal{L}(n)} p_l||\mathbf{v}_l||_2^2 \leq \;\; p_n^{\max} \\ & ||\mathbf{v}_l||_2 = 1, \quad p_l \geq 0, \; l \in \mathcal{L}(n) \end{array}$$

variables: $\{p_l, \mathbf{v}_l\}_{l\in\mathcal{L}(n)}$

master problem:

$$\begin{array}{ll} \text{minimize} & \sum_{n\in\mathcal{N}} f_n(\mathbf{z}) \\ \text{subject to} & \mathbf{z} \succeq \mathbf{0} \;\; , \\ \text{variables:} & \mathbf{z} = \{z_{nl}\}_{n\in\mathcal{N},\, l\in\mathcal{L}_{\mathrm{int}}(n)} \end{array}$$

Subproblem (BS optimization)



Subproblem

$$
\begin{aligned}
\text{minimize} \quad & -\sum_{l\in\mathcal{L}(n)} \beta_l \log(1+\gamma_l) \\
\text{subject to} \quad & \gamma_l \le \frac{p_l|\mathbf{h}_{ll}^{\mathrm{H}}\mathbf{v}_l|^2}{\sigma_l^2 + \sum\limits_{j\in\mathcal{L}(n), j\ne l} p_j|\mathbf{h}_{ll}^{\mathrm{H}}\mathbf{v}_j|^2 + \sum\limits_{i\in\mathcal{N}_{\mathrm{int}}(l)} z_{il}}, \; l\in\mathcal{L}(n) \\
& z_{il} \ge \sum_{j\in\mathcal{L}(i)} p_j|\mathbf{h}_{jl}^{\mathrm{H}}\mathbf{v}_j|^2, \; l\in\mathcal{L}_{\mathrm{int}}(n) \\
& \sum_{l\in\mathcal{L}(n)} p_l||\mathbf{v}_l||_2^2 \le p_n^{\max} \\
& ||\mathbf{v}_l||_2 = 1, \quad p_l \ge 0, \; l\in\mathcal{L}(n)
\end{aligned}
$$

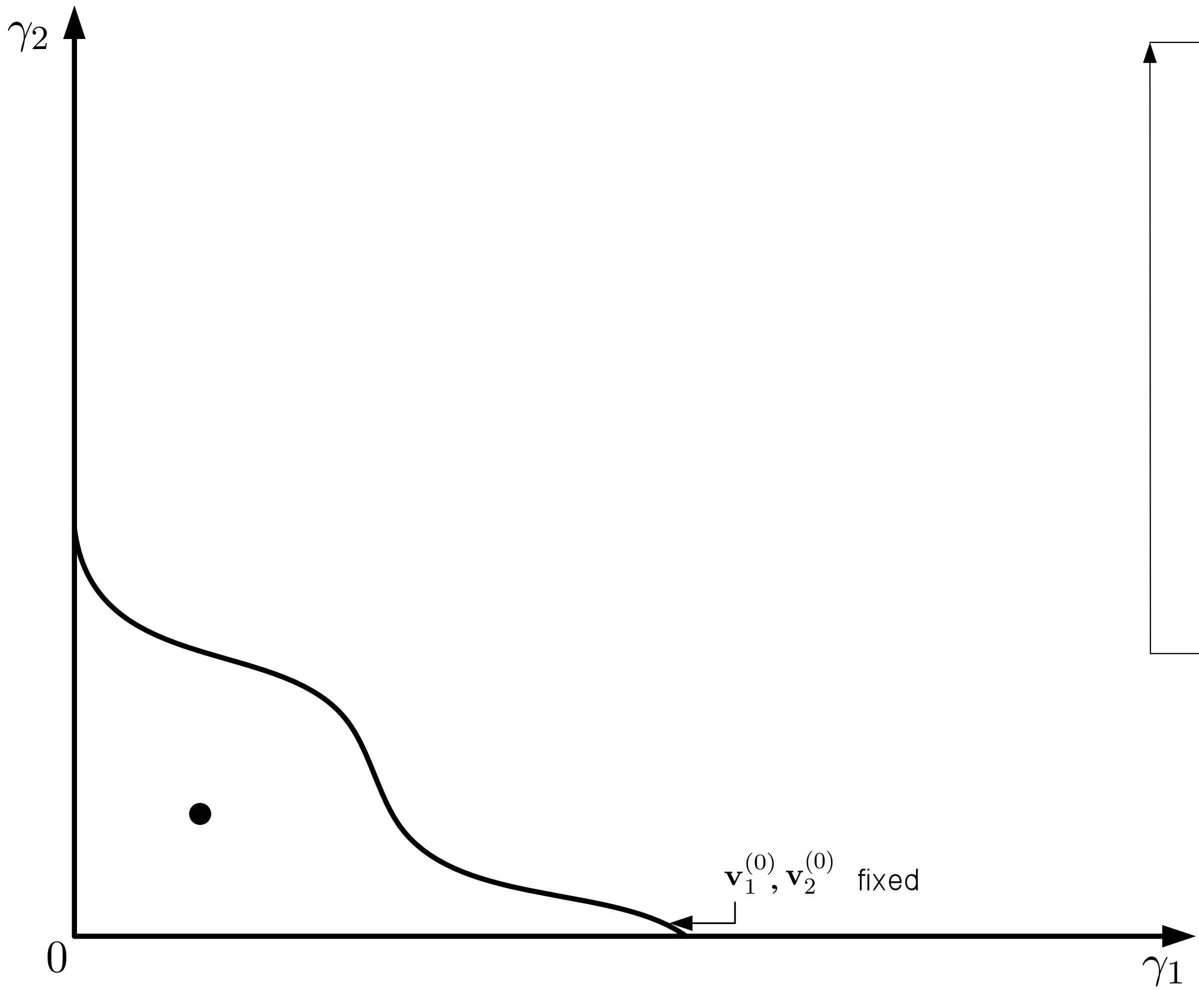
variables: $\{p_l, \gamma_l, \mathbf{v}_l\}_{l\in\mathcal{L}(n)}$

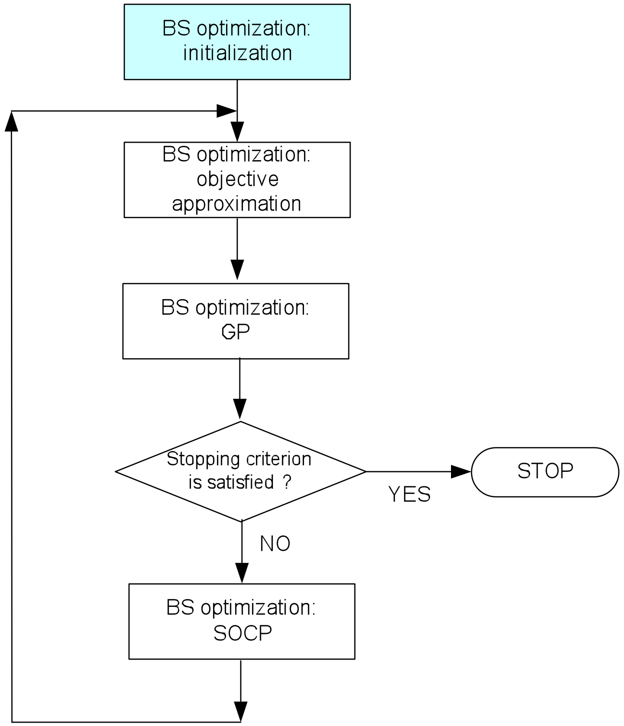
- The problem above is NP-hard
- Suboptimal methods, approximations

Subproblem: key idea

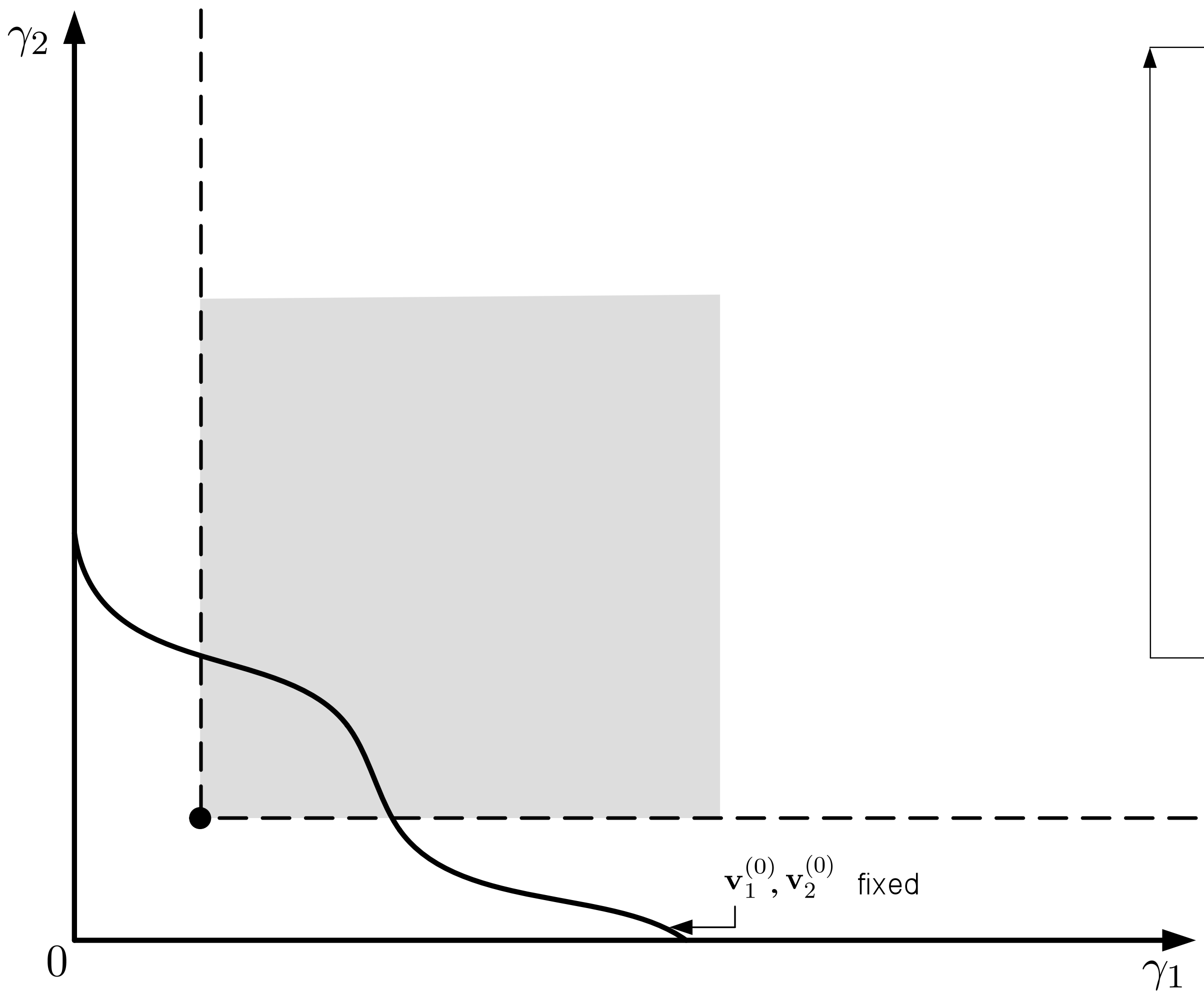
- The method is inspired from **alternating convex optimization** techniques
- Fix beamforming directions $\{\mathbf{v}_l\}_{l \in \mathcal{L}(n)}$
- Approximate objective $-\sum_{l \in \mathcal{L}(n)} \beta_l \log(1+\gamma_l)$ by an UB function
  - resultant problem is a **GP**; variables $\{p_l, \gamma_l\}_{l \in \mathcal{L}(n)}$
- Fix the resultant SINR values $\{\gamma_l\}_{l \in \mathcal{L}(n)}$
- Find beamforming directions $\{\mathbf{v}_l\}_{l \in \mathcal{L}(n)}$ that can preserve the SINR values with a **power margin**
  - this can be cast as a **SOCP**
- Iterate until a stopping criterion is satisfied

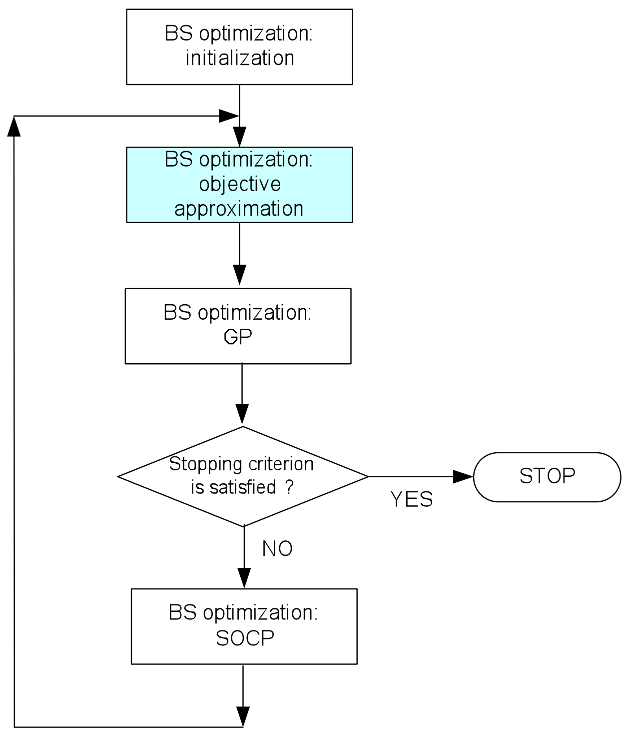
Subproblem: key idea



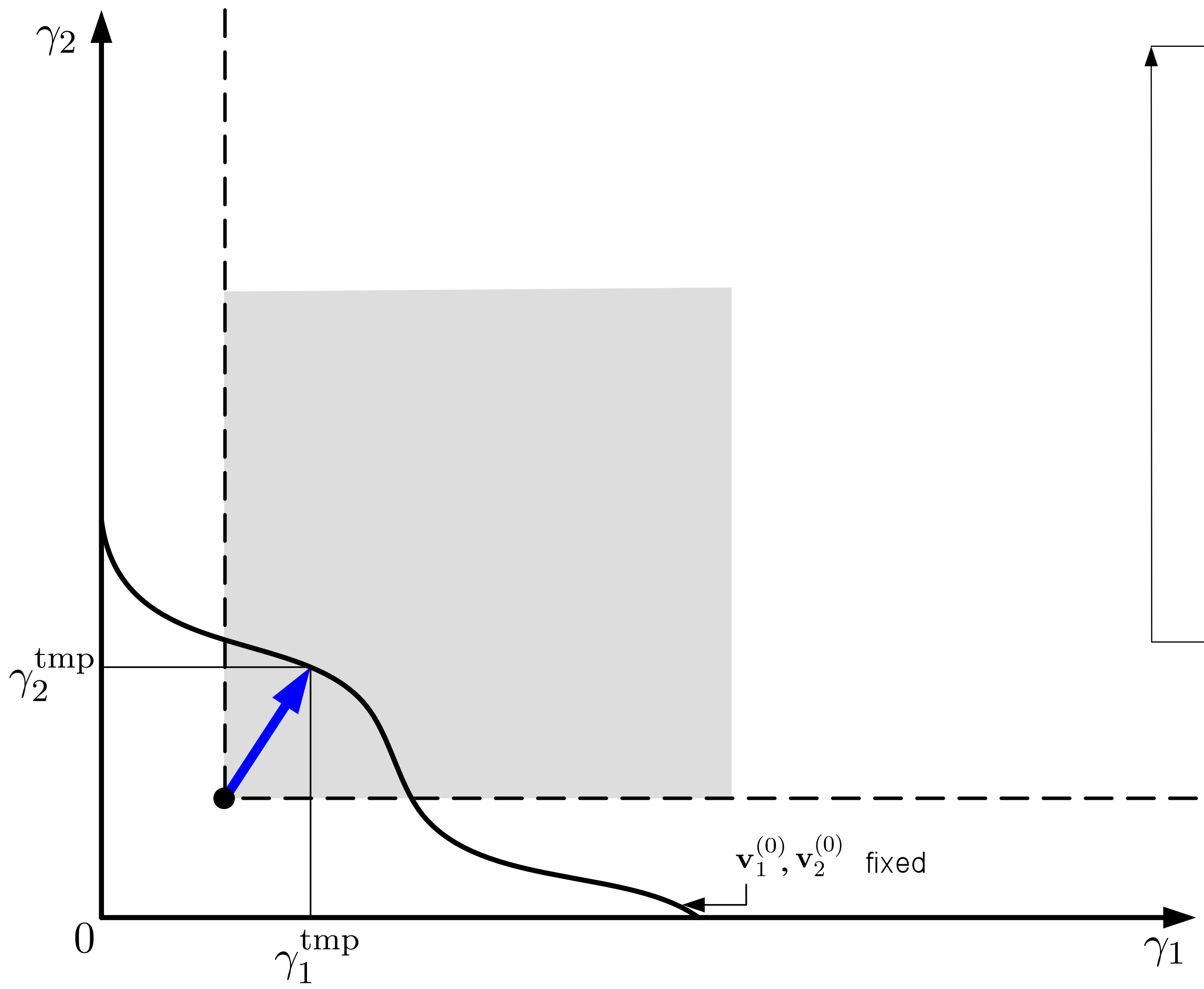


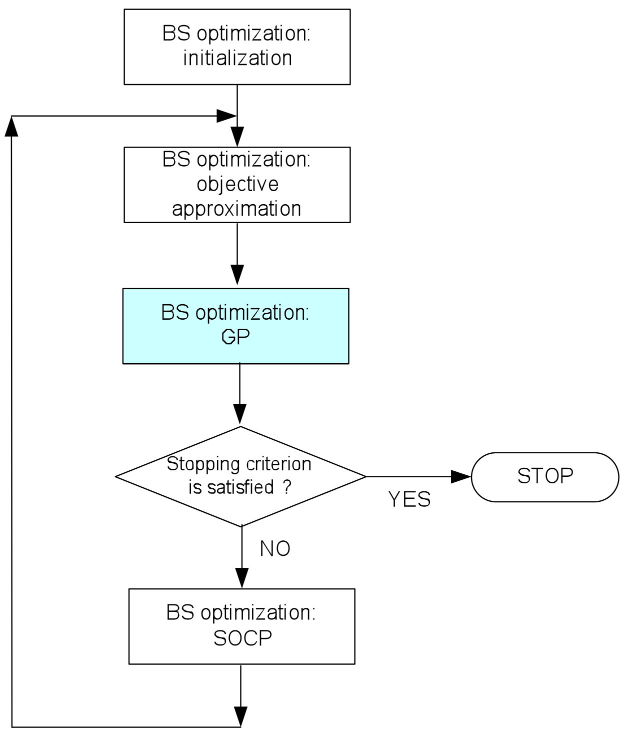
Subproblem: key idea



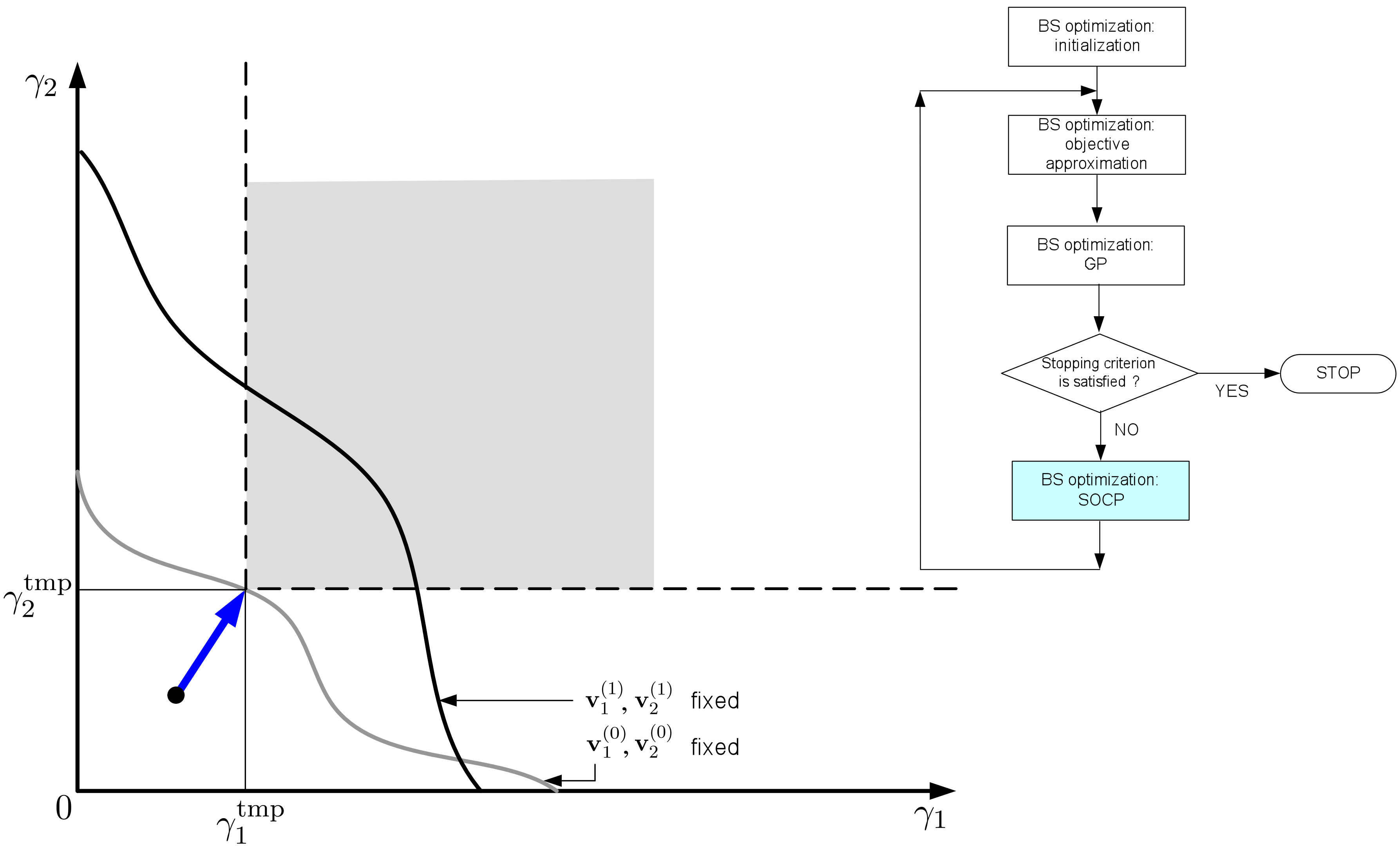


Subproblem: key idea

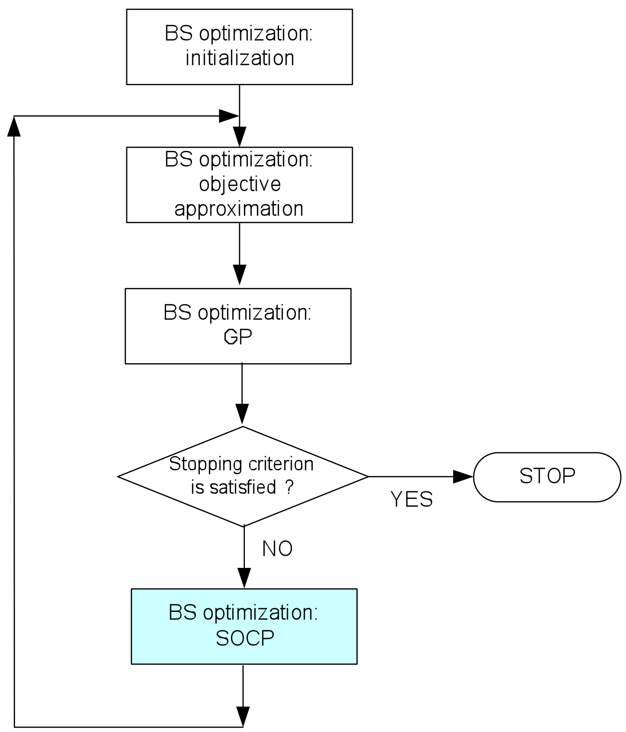




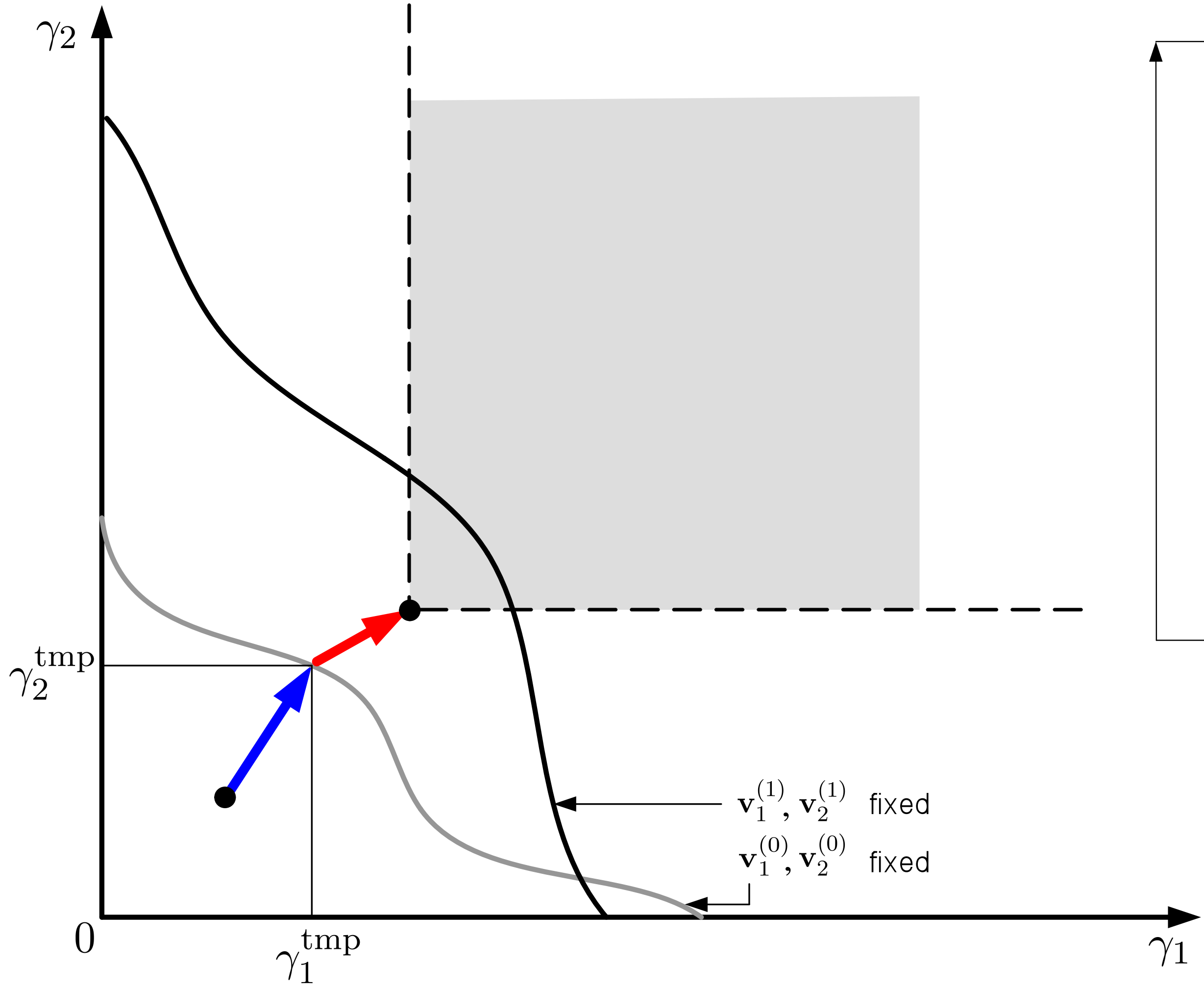
Subproblem: key idea

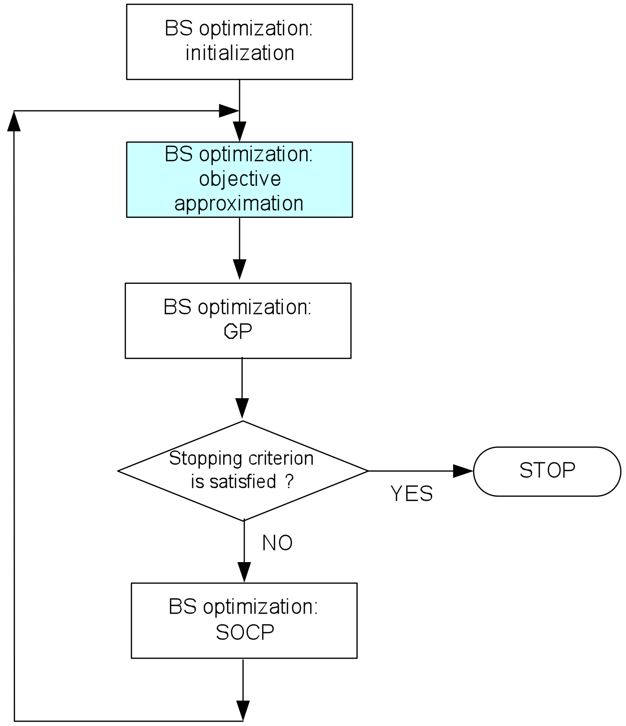


Subproblem: key idea

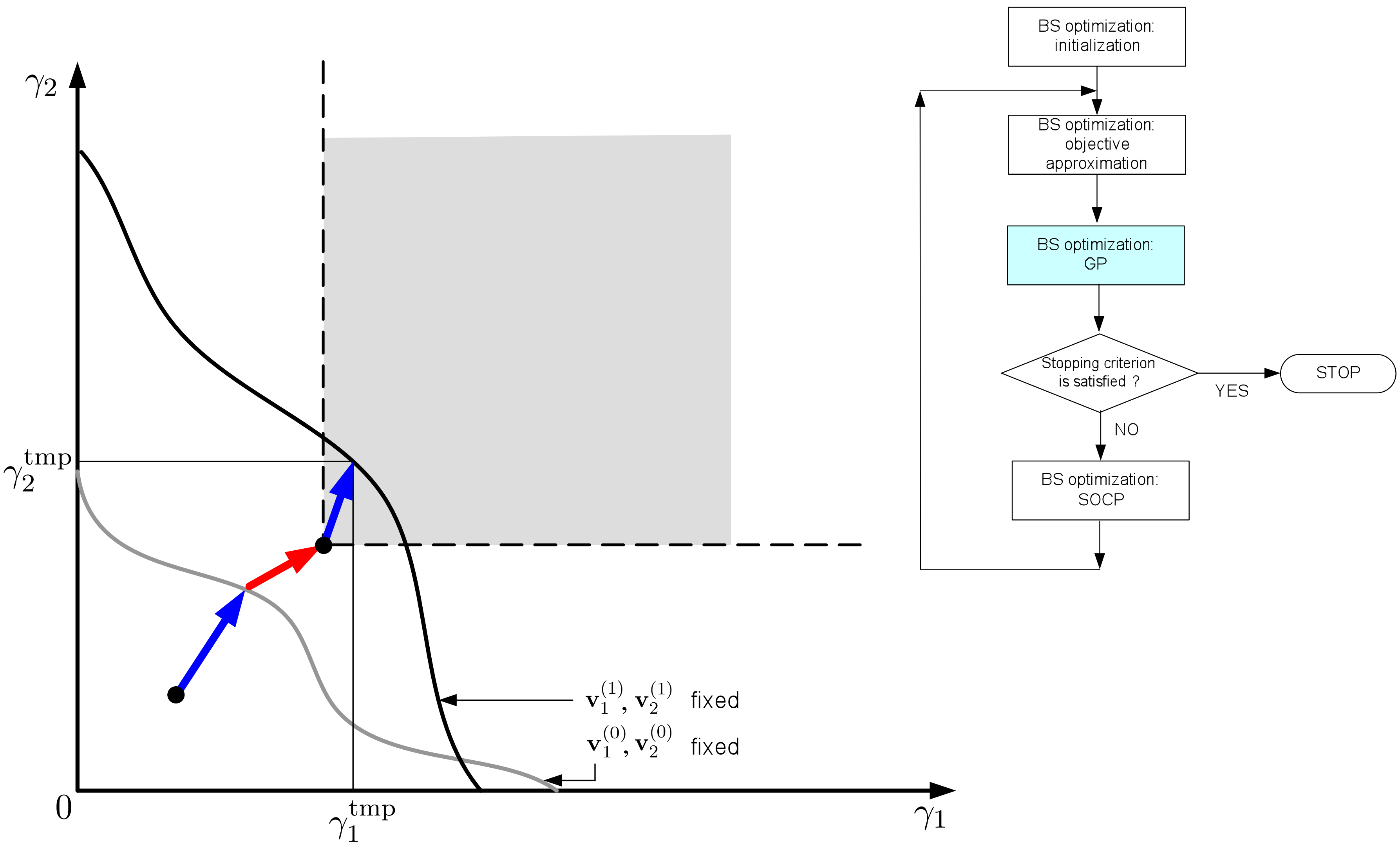


Subproblem: key idea

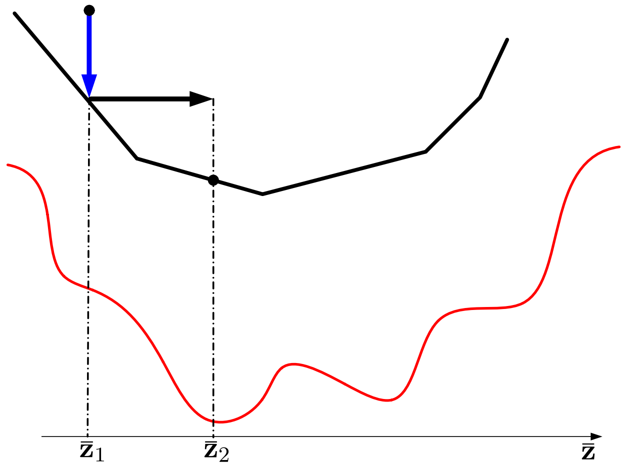




Subproblem: key idea



Master problem



Master problem

$$\begin{array}{ll} \text{minimize} & \sum_{n \in \mathcal{N}} f_n(\mathbf{z}) \\ \text{subject to} & \mathbf{z} \succeq \mathbf{0} \ , \\ \text{variables:} & \mathbf{z} = \{z_{nl}\}_{n \in \mathcal{N},\, l \in \mathcal{L}_{\text{int}}(n)} \end{array}$$

- Recall: subproblems are NP-hard -> we cannot even compute the master objective value
- Suboptimal methods, approximations
- Problem is nonconvex -> subgradient method alone fails

Master problem: key idea

- The method is inspired from **sequential convex approximation (upper bound) techniques**
- **subgradient method** is adopted to solve the resulting convex problems

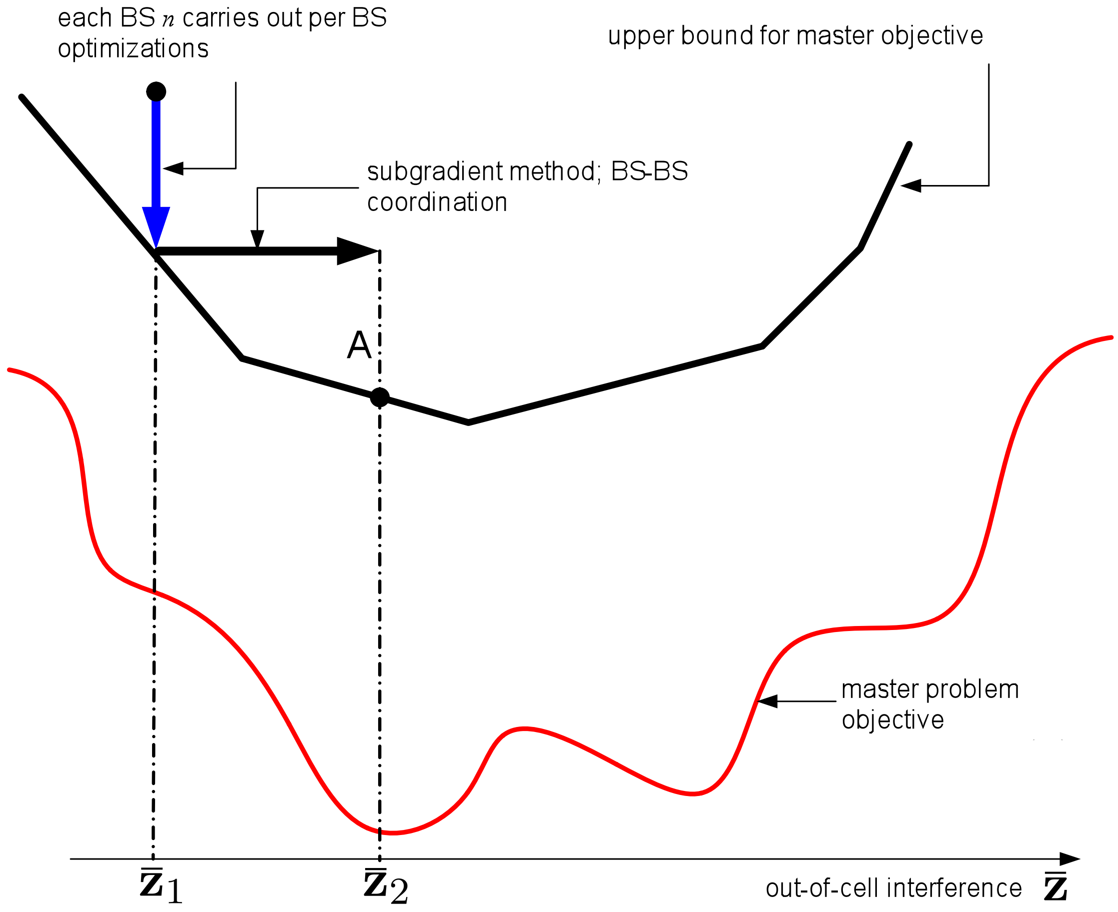
Integrate master problem & subproblem

- Increasingly important:

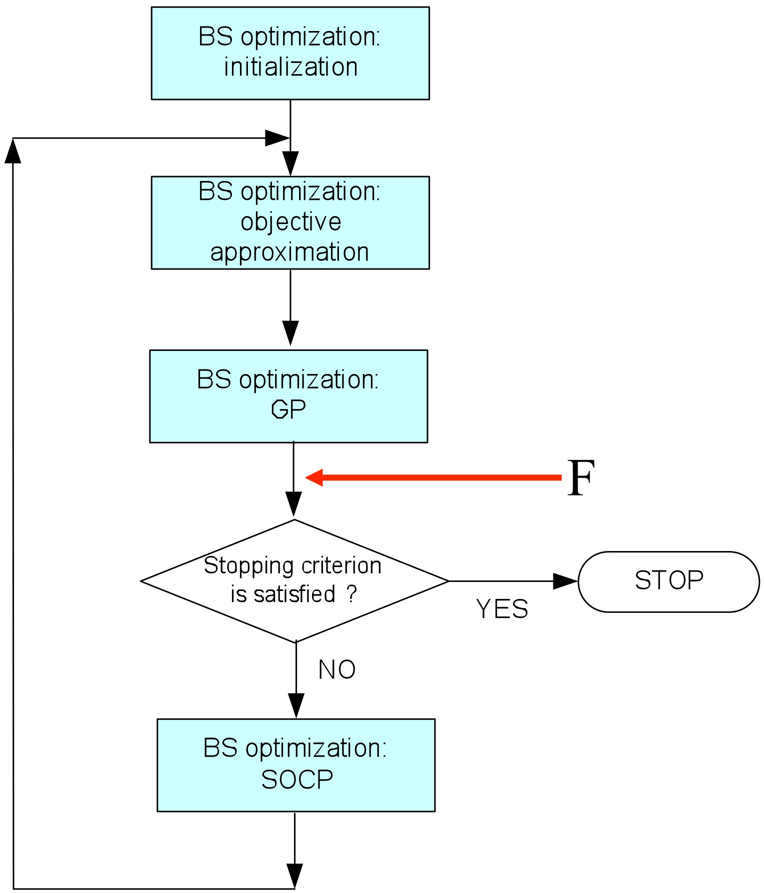
  - **convex approximations** mentioned above are such that we can always **rely on** the results of **BS optimizations** to compute a **subgradient** for the **subgradient method**.

  - thus, **coordination of the BS optimizations**

Integrate master problem & subproblem



Integrate master problem & subproblem



subproblem

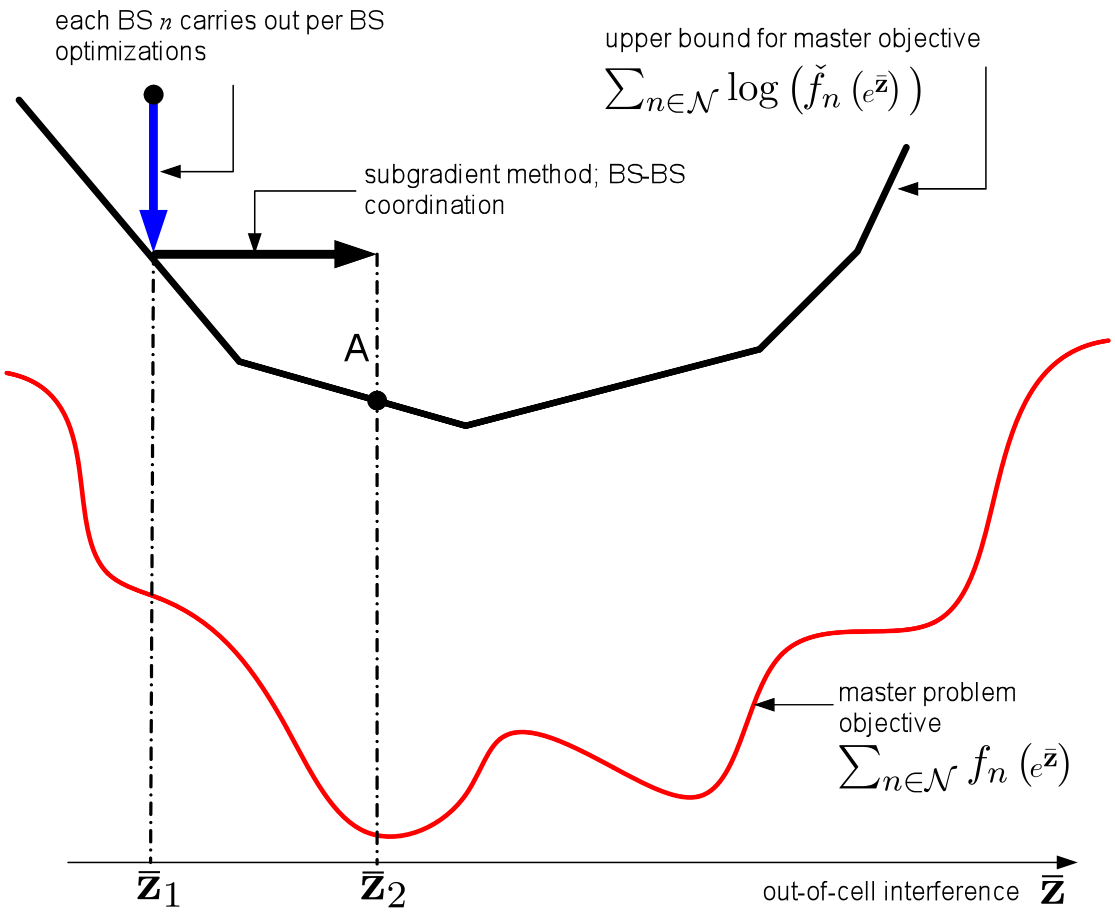
- out-of-cell interference: $\mathbf{z}$
- objective value computed by BS n at 'F' : $\check{f}_n(\mathbf{z})$
- we can show that

$$\sum_{n\in\mathcal{N}} f_n(\mathbf{z}) \leq \sum_{n\in\mathcal{N}} \log\left(\check{f}_n(\mathbf{z})\right)$$

$$\sum_{n\in\mathcal{N}} f_n(e^{\bar{\mathbf{z}}}) \leq \underbrace{\sum_{n\in\mathcal{N}} \log\left(\check{f}_n(e^{\bar{\mathbf{z}}})\right)}_{\text{convex}}$$

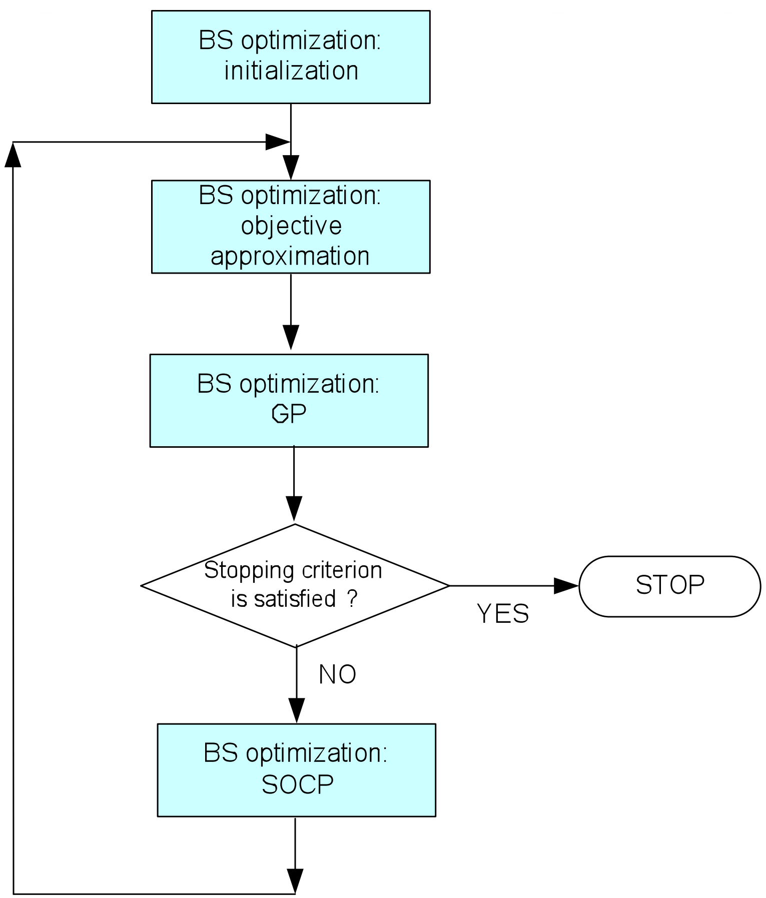
- **optimal sensitivity values of GP -> construct subgradient**

Integrate master problem & subproblem

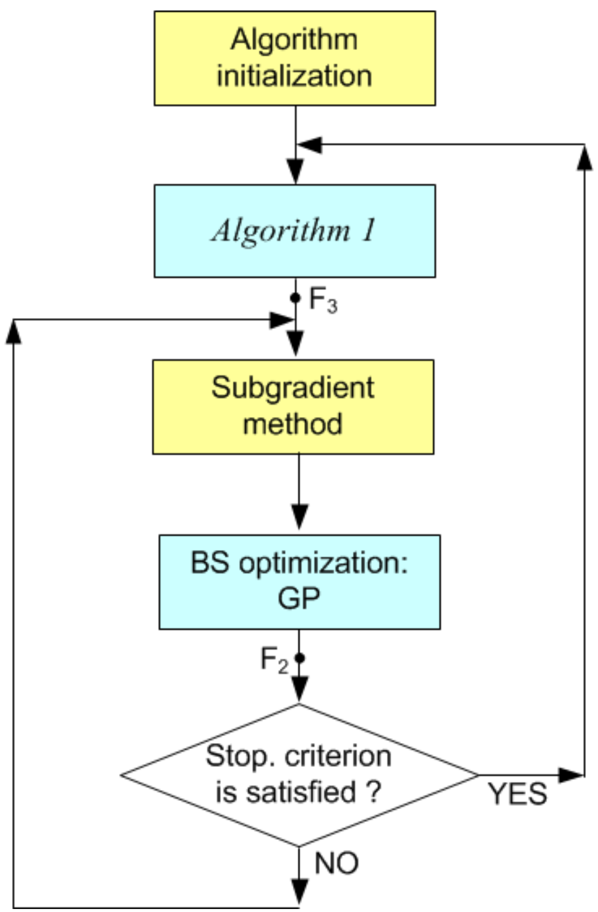


subgradient method: $\bar{z}_{il}^{(j+1)} = \bar{z}_{il}^{(j)} - \theta^{(j)} \sum_{n\in\mathcal{N}} d_{il}^{n}(\bar{\mathbf{z}}^{(j)})$

Integrate master problem & subproblem

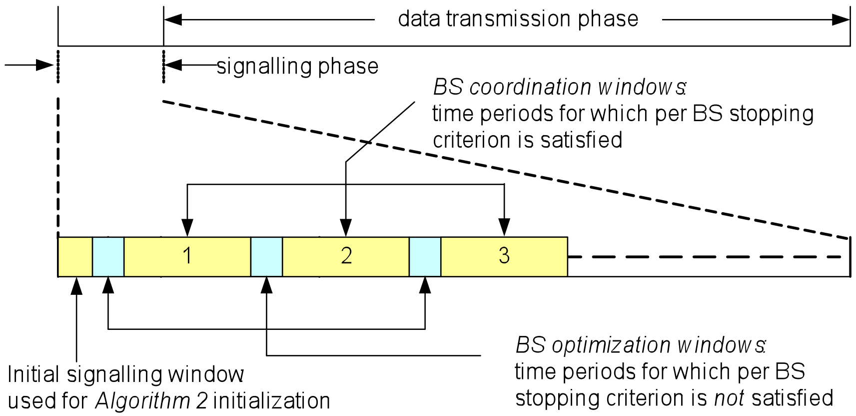


Algorithm 1
(subproblem)



Algorithm 2
(overall problem)

An example signaling frame structure

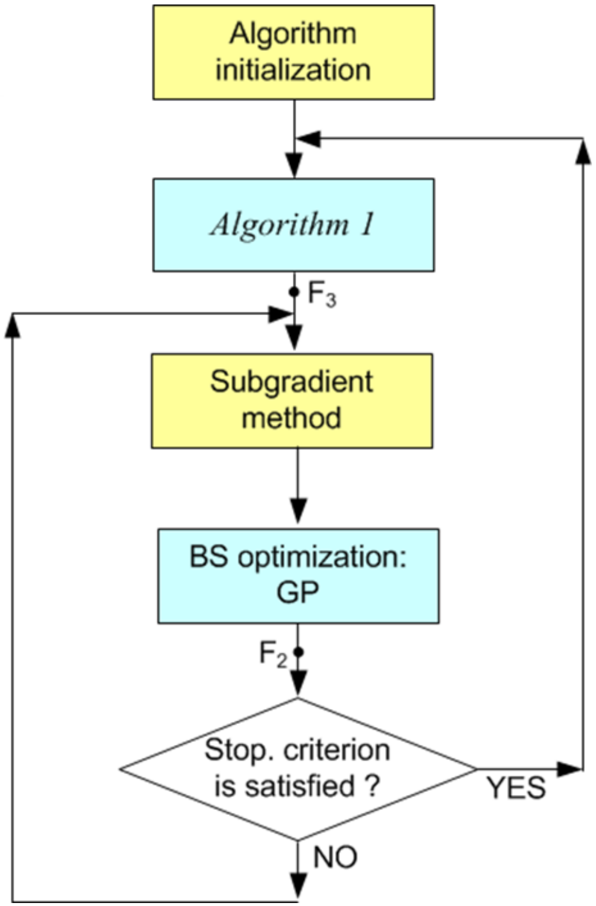


note: in *Alg.1* or subgradient method, 'BS optimization GP' is always carried out

in our simulations:

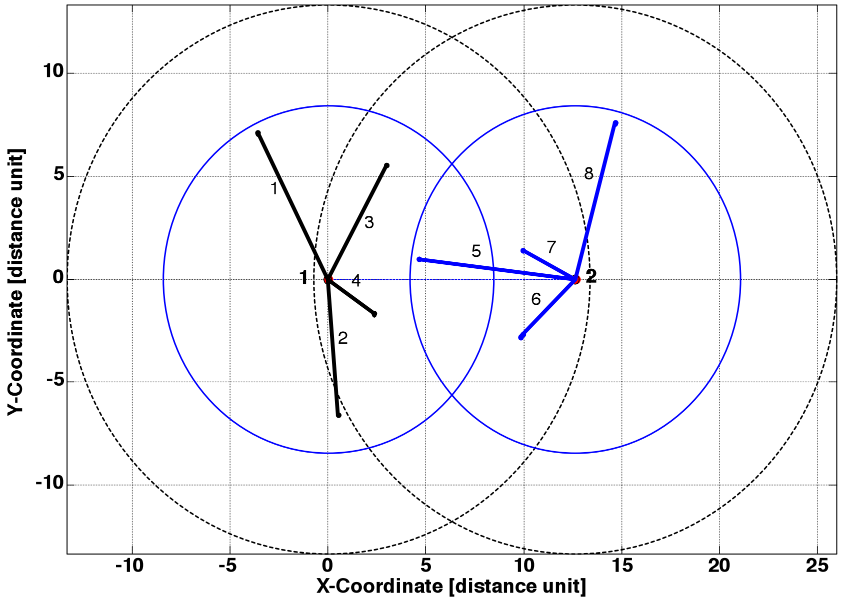
- fixed Alg.1 iterations ($J_{\mathrm{BS-opt}}$)

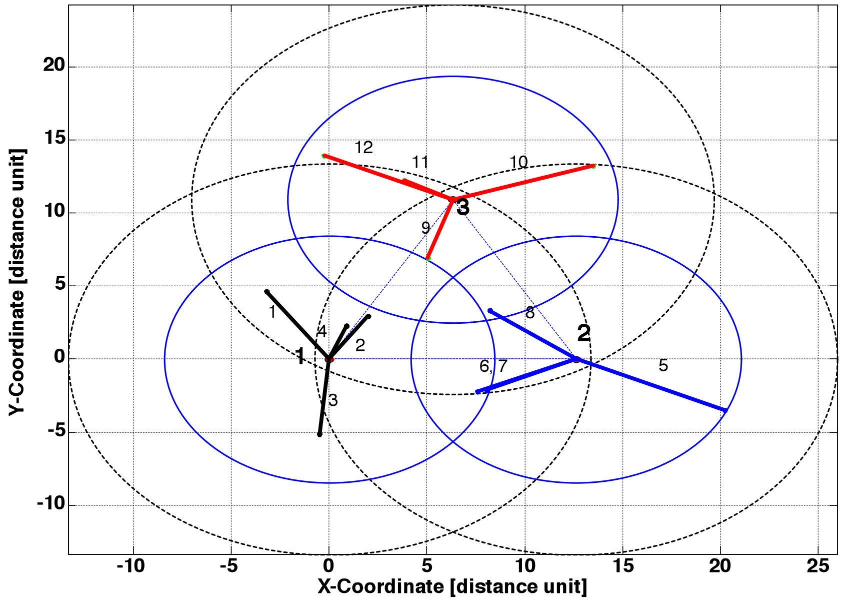
- fixed subgrad iterations (J_{subgrad}) per switch



Algorithm 2
(overall problem)

Numerical Examples





channel gains: $\mathbf{h}_{ij} = \sqrt{d_{ij}^{-4}}\ \mathbf{c}_{ij}$

d_{ij} : distance from $tran(i)$ to $rec(j)$
$\mathbf{c}_{ij}$: small scale fading coefficients

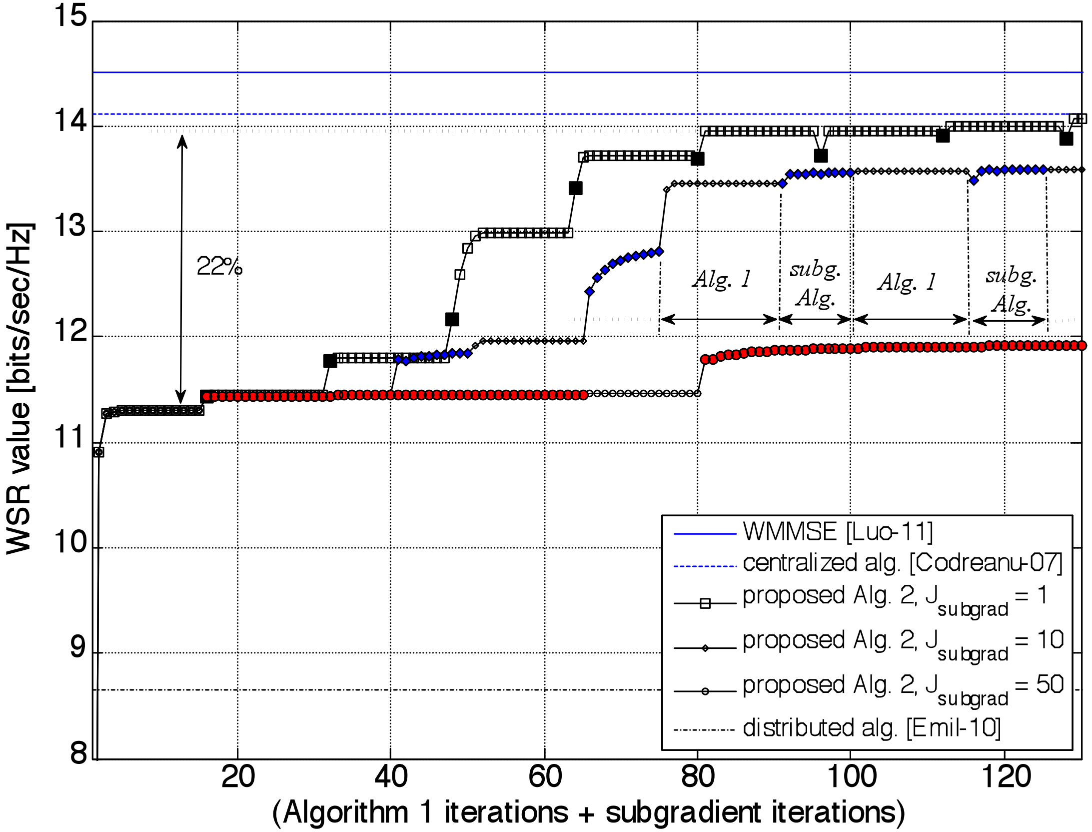
SNR operating point:

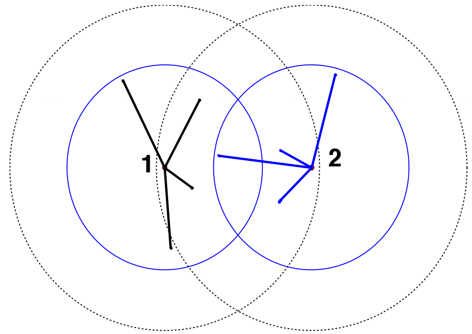
$$\mathrm{SNR}(d) = \begin{cases} \dfrac{p_0^{\max}}{\sigma_0^2} & d \leq 1 \\ \dfrac{p_0^{\max}}{\sigma_0^2}\, d^{-4} & \text{otherwise} \end{cases}$$

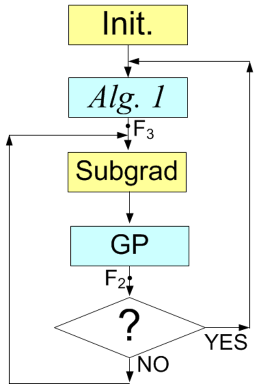
$p_0^{\max}/\sigma_0^2 = 45\mathrm{dB},\ \ D_{\mathrm{BS}} = 1.5\, R_{\mathrm{BS}}$

$\mathrm{SNR}(R_{\mathrm{BS}}) = 8\mathrm{dB},\ \ \mathrm{SNR}(R_{\mathrm{int}}) = 0\mathrm{dB}$

Numerical Examples

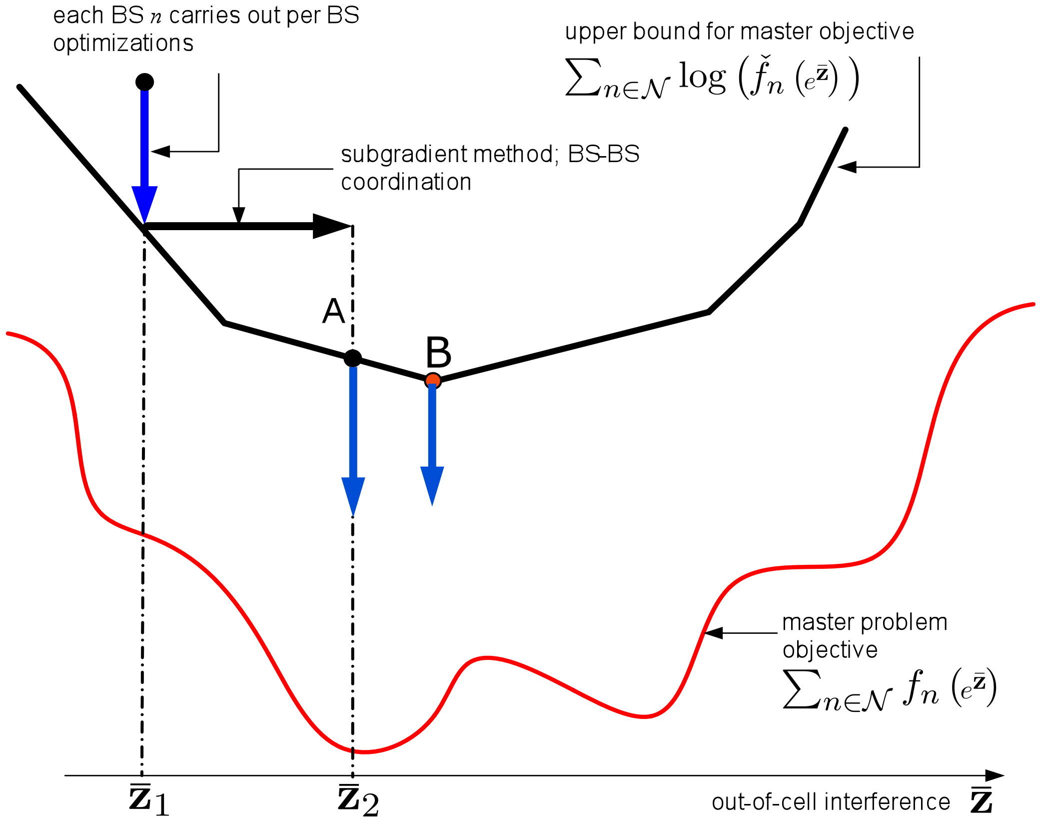






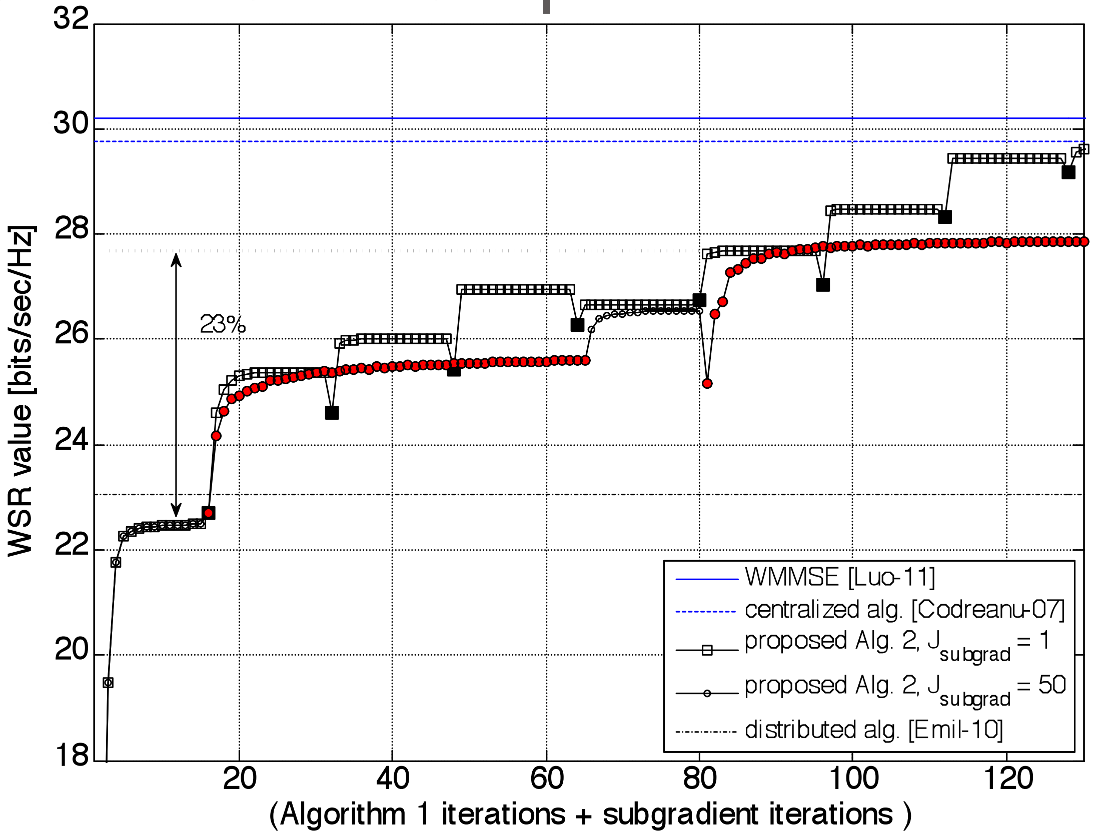
- J_{subgrad} -> the degree of BS coordination
- note -> subgradient is not an ascent method
- out-of-cell interference is resolved -> objective value is increased
- **smaller** J_{subgrad} performs **better** compared to large J_{subgrad}
- **light backhaul signaling**

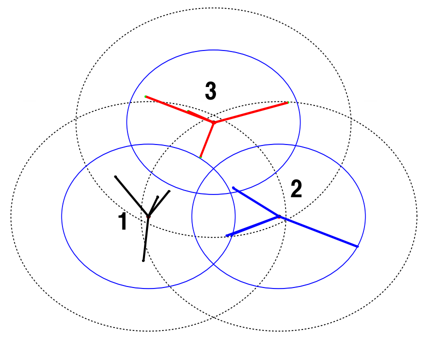
Numerical Examples

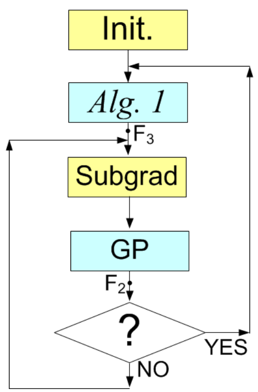


- **accuracy** of the solution of an **approximated master problem** is **irrelevant** in the case of overall algorithm
- refining the approximation more often is more beneficial

Numerical Examples

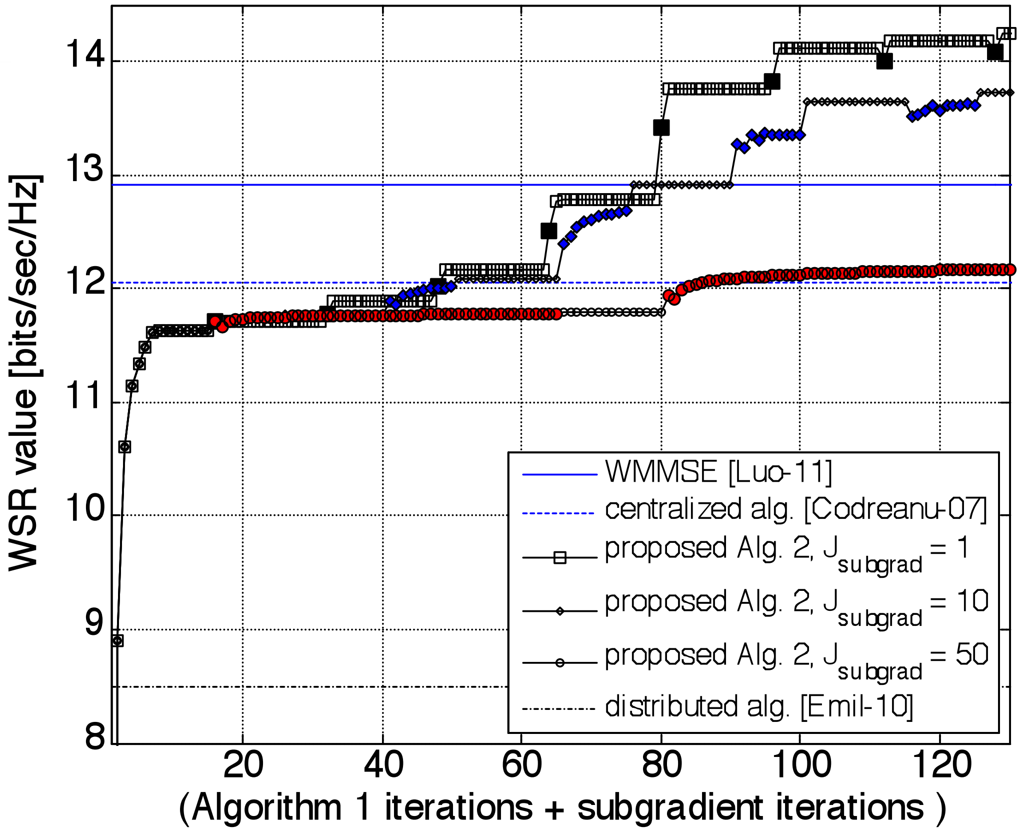


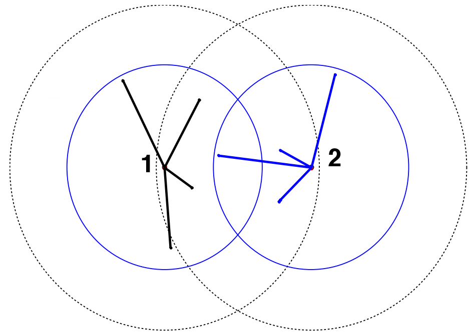


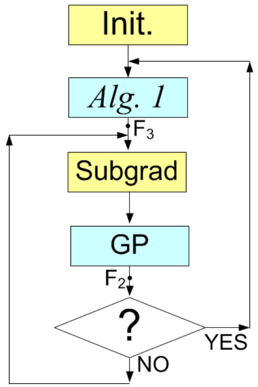


- same behavior
- smaller J_{subgrad} performs better compared to large

Numerical Examples

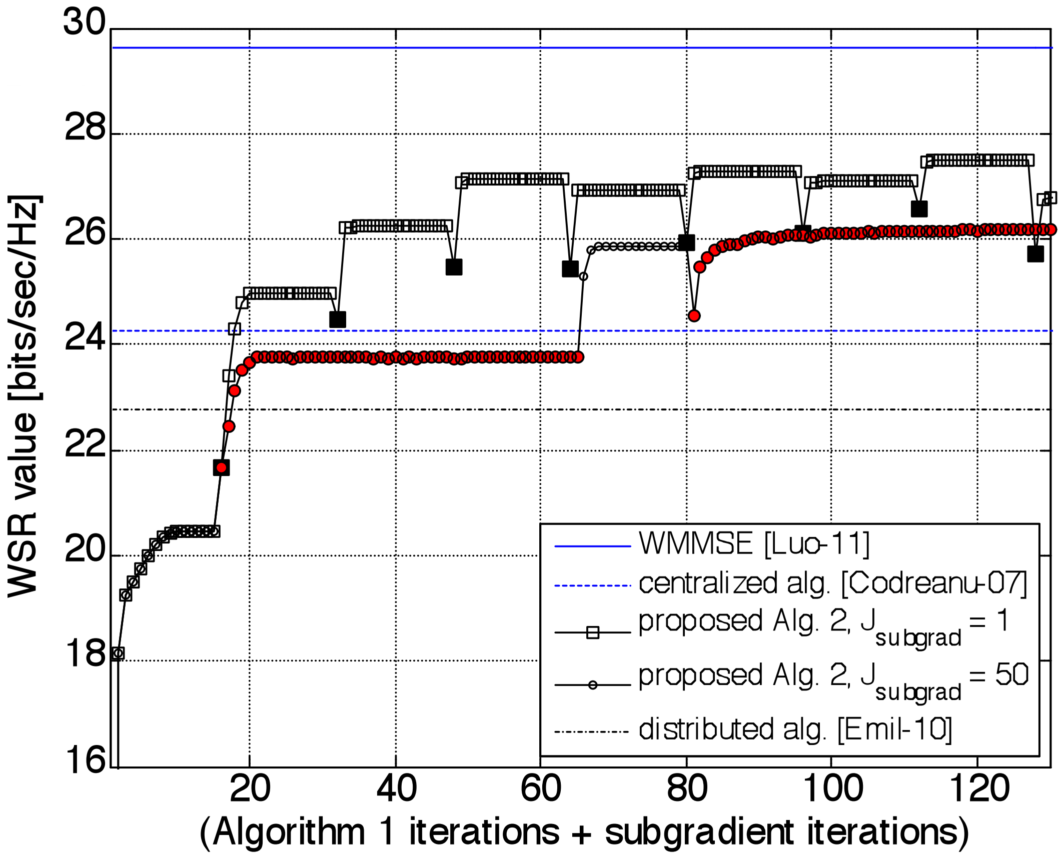


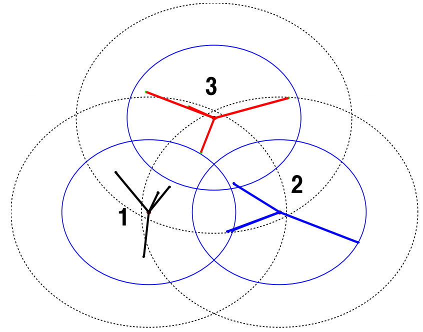


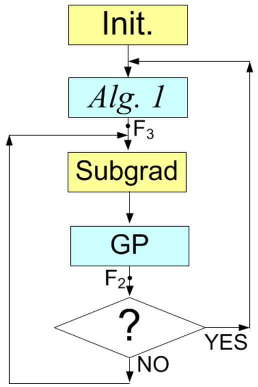


- algorithm performs better than the centralized algorithm
- not surprising since both algorithms are suboptimal algorithms

Numerical Examples

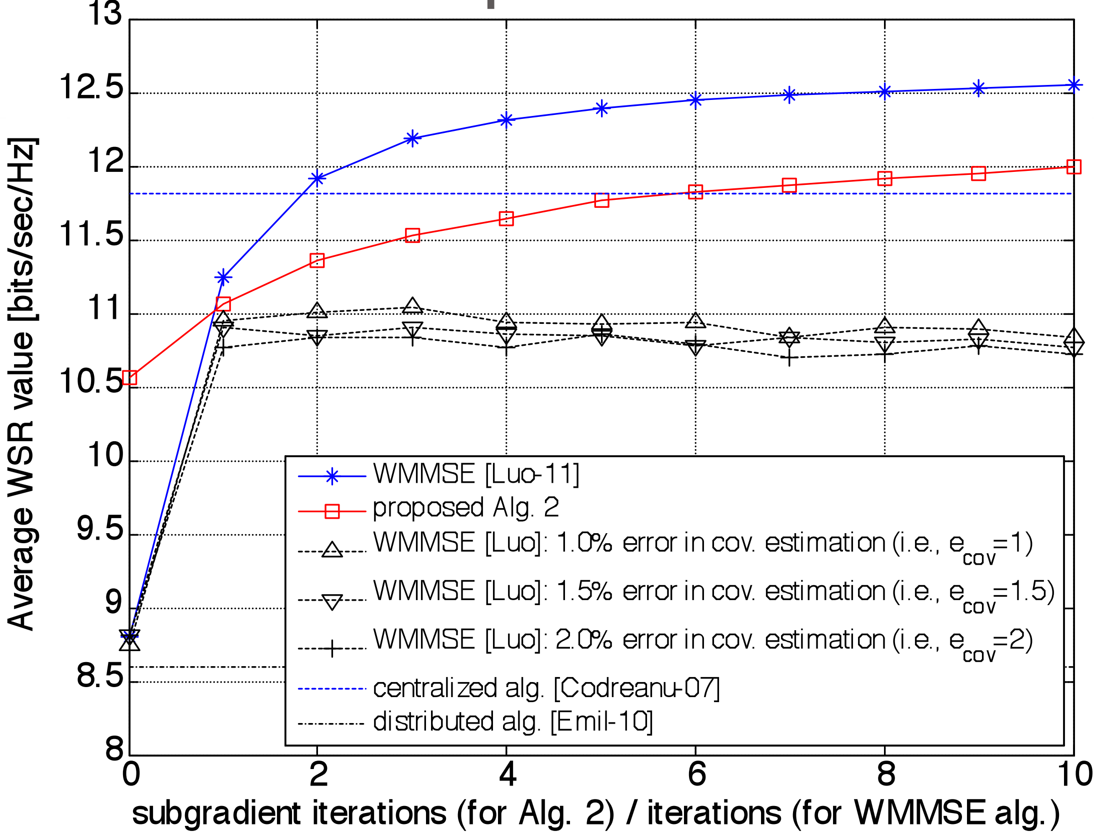


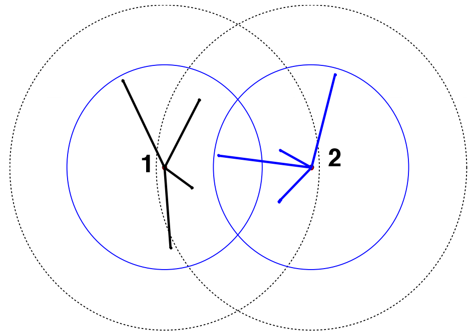


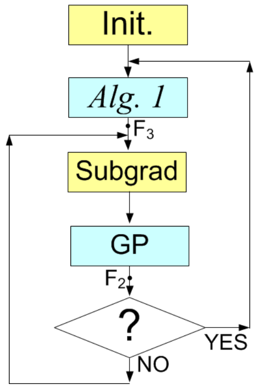


- algorithm performs better than the centralized algorithm
- not surprising since both algorithms are suboptimal algorithms

Numerical Examples

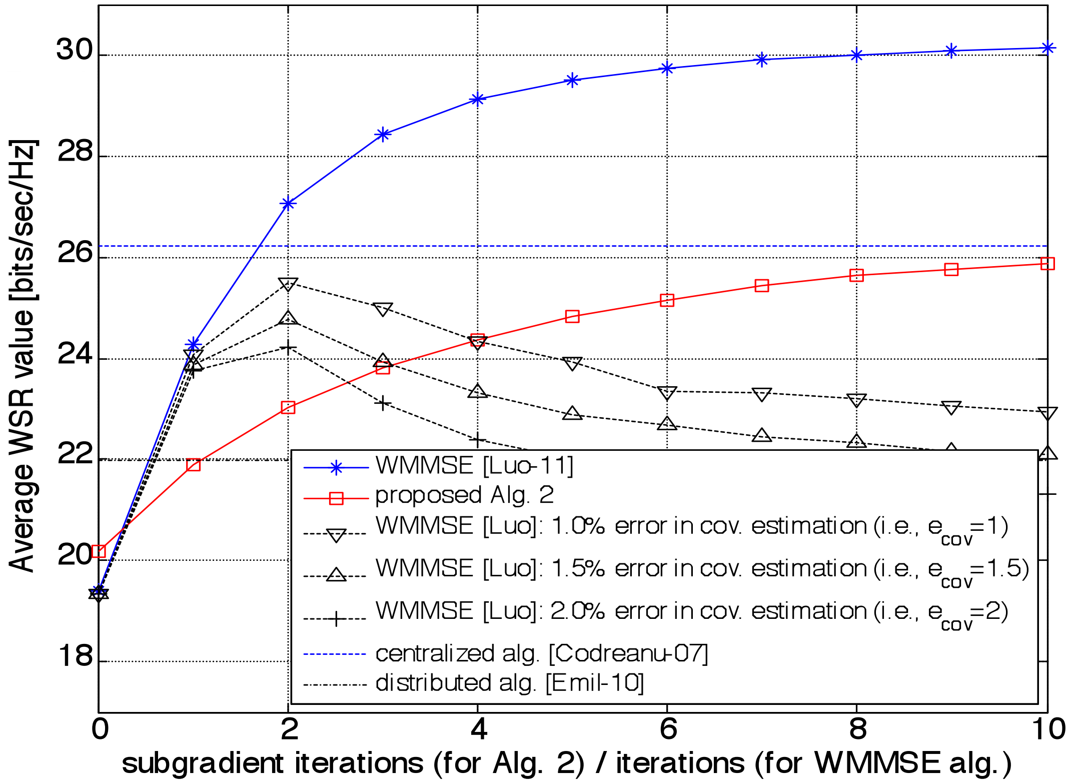


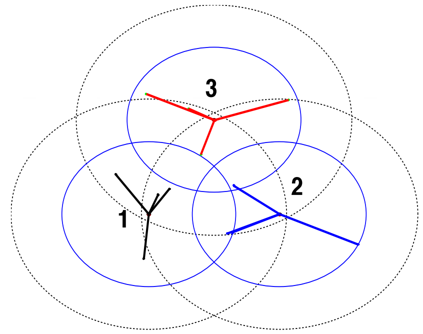


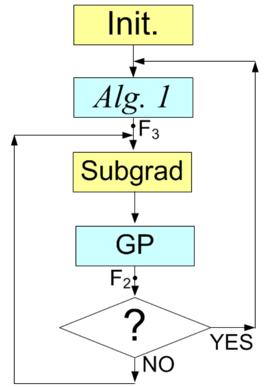


- $J_{\text{subgrad}} = 1$; **one subgradient iteration during BS coordination window**
- **12%** improvement **within 5 BS coordination**
- **99%** of the centralized value **within 5 BS coordination**
- WMMSE performs better with no errors in user estimates
- WMMSE with even 1% error in signal covariance estimations at user perform poorly

Numerical Examples







- $J_{\mathrm{subgrad}} = 1$; **one subgradient iteration during BS coordination window**
- **24%** improvement **within 5 BS coordination**
- **94%** of the centralized value **within 5 BS coordination**
- WMMSE performs better with no errors in user estimates
- WMMSE with even 1% error in signal covariance estimations at user perform poorly

Conclusions

- **Problem:** WSRMax for MISO interfering BC channel (NP-hard)
- **Techniques:** Primal decomposition
- **Result:** many subproblems (one for each BS) coordinating to find a suboptimal solution of the original problem
- **Subproblem:** alternating convex approximation techniques, GP, and SOCP
- **Master problem:** sequential convex approximation techniques and subgradient method
- **Coordination:** BS-BS (backhaul) signaling; **favorable for practical implementation**
- Substantial improvements with a small number of BS coordination -> **favorable for practical implementation**
- **Numerical examples** -> algorithm performance is significantly close to (suboptimal) centralized solution methods