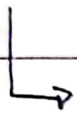




$$\therefore Q(\phi' | \theta) = L(\phi') + H(\phi' | \theta)$$

①

$$H(\phi' | \theta) \leq H(\phi | \phi)$$



$$H(\phi' | \theta) = \mathbb{E} \left\{ \log \kappa(x|y, \phi') \mid y, \theta \right\}$$

$$= \int \log \kappa(x|y, \phi') \kappa(x|y, \theta) dx$$

$$= - \int \log [\kappa(x|y, \phi')]^{-1} \kappa(x|y, \theta) dx$$

$$= - \int \log \left[ \frac{\kappa(x|y, \theta)}{\kappa(x|y, \phi')} \times \frac{1}{\kappa(x|y, \theta)} \right] \kappa(x|y, \theta) dx$$

$$= - \int \left[ \log \left( \frac{\kappa(x|y, \theta)}{\kappa(x|y, \phi')} \right) - \log \kappa(x|y, \theta) \right] \kappa(x|y, \theta) dx$$

$$= \underbrace{\int \log \kappa(x|y, \theta) \kappa(x|y, \theta) dx}_{H(\phi | \phi)} - \underbrace{\int \log \left[ \frac{\kappa(x|y, \theta)}{\kappa(x|y, \phi')} \right] \kappa(x|y, \theta) dx}_{D_{KL} \left[ \frac{\kappa(x|y, \theta)}{\kappa(x|y, \phi')} \right]}$$

$$H(\phi' | \theta) = H(\phi | \phi) - D_{KL} \left[ \frac{\kappa(x|y, \theta)}{\kappa(x|y, \phi')} \right]$$

$$\leq H(\phi | \phi)$$

$$\therefore H(\phi'|\phi) \leq H(\phi|\phi)$$

with equality if and only if

$$K(x|y, \phi') = K(x|y, \phi)$$

almost everywhere

From ① we have

$$Q(\phi'|\phi) = L(\phi') + H(\phi'|\phi)$$

$$\begin{aligned} Q(\phi'|\phi) - L(\phi') &= H(\phi'|\phi) \\ &\leq H(\phi|\phi) \end{aligned}$$

$$\therefore Q(\phi'|\phi) - L(\phi') \leq H(\phi|\phi)$$

Equality holds when

$$\phi' = \phi$$