- Related to page 9 of L5
- Please replace ϕ' by θ and ϕ by $\theta^{(n)}$ to match the notation with that of Lectures.

$$Q(\phi'|\phi) = \mathbb{E}\{\log r(x|\phi') \mid y, \phi\}$$

$$\boxed{\begin{array}{ll} \text{E-Step:} & \text{Compute } Q(\phi|\phi^{(p)}) \\ \text{M-Step:} & \phi^{(p+1)} = \underset{\phi}{\arg\max}\, Q(\phi|\phi^{(p)}) \end{array}}$$

$$H(\phi'|\phi) = \mathbb{E}\{\log k(x|y,\phi') \mid y, \phi\}$$

(1) $L(\phi) = \log\{s(y|\phi)\}$

(2) $k(x|y,\phi) = r(x|\phi)/s(y|\phi)$

(3) $Q(\phi'|\phi) = \mathbb{E}\{\log r(x|\phi') \mid y, \phi\}$

$$H(\phi'|\phi) = \mathbb{E}\left\{\log\left[\frac{r(x|\phi')}{s(y|\phi')}\right] \middle| y, \phi\right\}$$

$$= \underbrace{\mathbb{E}\{\log r(x|\phi') \mid y,\phi\}} - \underbrace{\mathbb{E}\{\log s(y|\phi') \mid y, \phi\}}$$

$$= Q(\phi'|\phi) - \underbrace{\log s(y|\phi')}_{L(\phi')}$$

$$\therefore \; Q(\phi' \mid \phi) = L(\phi') + H(\phi' \mid \phi) \quad (1)$$

$$H(\phi' \mid \phi) \leqslant H(\phi \mid \phi)$$

$$H(\phi' \mid \phi) = \mathbb{E}\left\{ \log K(x \mid y, \phi') \,\middle|\, y, \phi \right\}$$

$$= \int \log K(x \mid y, \phi')\, K(x \mid y, \phi)\, dx$$

$$= -\int \log \left[K(x \mid y, \phi')\right]^{-1} K(x \mid y, \phi)\, dx$$

$$= -\int \log \left[\frac{K(x \mid y, \phi)}{K(x \mid y, \phi')} \times \frac{1}{K(x \mid y, \phi)} \right] K(x \mid y, \phi)\, dx$$

$$= -\int \left[\log \left(\frac{K(x \mid y, \phi)}{K(x \mid y, \phi')} \right) - \log K(x \mid y, \phi) \right] K(x \mid y, \phi)\, dx$$

$$= \underbrace{\int \log (K \mid y, \phi)\, K(x \mid y, \phi)\, dx}_{H(\phi \mid \phi)} - \underbrace{\int \log \left[\frac{K(x \mid y, \phi)}{K(x \mid y, \phi')} \right] K(x \mid y, \phi)\, dx}_{D_{KL}}$$

$$H(\phi' \mid \phi) = H(\phi \mid \phi) - D_{KL}\left[\frac{K(x \mid y, \phi)}{K(x \mid y, \phi')} \right]$$

$$\leqslant H(\phi \mid \phi)$$

$\therefore \quad H(\phi'|\phi) \leq H(\phi|\phi)$

with equality if and only if

$K(x|y,\phi') = K(x|y,\phi)$

almost everywhere

From ① we have

$$Q(\phi'|\phi) = L(\phi') + H(\phi'|\phi)$$

$$Q(\phi'|\phi) - L(\phi') = H(\phi'|\phi)$$
$$\leq H(\phi|\phi)$$

$$\therefore \quad Q(\phi'|\phi) - L(\phi') \leq H(\phi|\phi)$$

Equality holds when

$\phi' = \phi$