MM Optimization Algorithms
Homework 3

Problem 1

Suppose you are given a linear system

$$Ax = b, \tag{1}$$

where $A \in \mathbb{R}^{m\times n}$, $x \in \mathbb{R}^n$, and $b \in \mathbb{R}^m$. Moreover, $\texttt{rank}\ A = r < m < n$. It is known that $b \in \texttt{range}\ A$. If a scalar $L > \|A\|$ is given, determine a method to compute a solution of the linear system by using the MM proncipole.

Problem 2

1. Obtain a minorization function to f

$$f(x) = 1 + x \tag{2}$$

   at $x^{(n)} \in \mathbb{R}_{++}$ where the minorization $g(\,\cdot\,|x^{(n)})$ is of the form

$$g(x|x^{(n)}) = mx^r \tag{3}$$

   for some m and r that depends on $x^{(n)}$, which are to determined.

2. By using the above minorization apply the MM principle to derive an algorithm to find a solution [1] to the problem

$$\begin{array}{ll} \text{maximize} & \log\left(1 + \dfrac{\alpha_1 p_1}{\sigma_1^2 + \beta_1 p_2}\right) + \log\left(1 + \dfrac{\alpha_2 p_2}{\sigma_2^2 + \beta_2 p_1}\right) \\ \text{subject to} & p_1 + p_2 \le p_{\texttt{tot}} \\ & p_1, p_2 > 0 \end{array}$$

   where the decision variables are p_1 and p_2 and $\alpha_1, \alpha_2, \beta_1, \beta_2, \sigma_1$, and σ_2 are positive scalars and are given.

   - Hint 1: You may use the substition $x_1 = \alpha_1 p_1/(\sigma_1^2 + \beta_1 p_2)$ and $x_2 = \alpha_2 p_2/(\sigma_2^2 + \beta_2 p_1)$.
   - Hint 2: You may use that log is an increasing finction.
   - Hint 3: The maximization of the minorization function subject to relevant constraints may be posed equaivalently as a GP (geometric program).

[1] The solution given by the algorithm is not necessarily optimal.