MM Optimization Algorithms
Homework 2

Problem 1

Let $a = [a_1, \ldots, a_N]^{\mathrm{T}}$ and $x = [x_1, \ldots, x_N]^{\mathrm{T}}$ be vectors in $\mathbb{R}^N$ with positive elements. Demonstrate the majorization

$$\frac{1}{a^{\mathrm{T}}x} \leq \frac{1}{\left(a^{\mathrm{T}}x^{(n)}\right)^2} \sum_{i=1}^{N} \frac{a_i x_i^{(n)2}}{x_i} \quad \text{for all } x \in \mathbb{R}_{++}^N \tag{1}$$

Problem 2

Suppose $f : \mathbb{R}^N \to \mathbb{R}$ is a convex function. Derive the majorization

$$f\left(\sum_{i=1}^{M} x_i\right) \leq \frac{1}{M} \sum_{i=1}^{M} f\left(M\left(x_i - x_i^{(n)} + \bar{x}^{(n)}\right)\right) \quad \text{for all } [x_1^{\mathrm{T}}, \ldots, x_M^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{MN} \tag{2}$$

where $\bar{x}^{(n)} = (1/M) \sum_{i=1}^{M} x_i^{(n)}$.

Problem 3

Obtain a minorization function for the problem given on page 32 of Lecture 2 (slides). Derive the corresponding MM algorithm explicitly identifying the related optimization problem for surrogate function maximization.