MM Optimization Algorithms
Homework 1

Problem 1

The fact that $x^2 \geq 0$ for all $x \in \mathbb{R}$ is commonly used when deriving many useful majorizing functions. By using this trick, show that

$$|\theta| \leq \frac{1}{2|\theta^{(n)}|}\,\theta^2 + \frac{1}{2}\,|\theta^{(n)}| \quad \text{for all } \theta \in \mathbb{R} \setminus \{0\} \tag{1}$$

$$||\theta|| \leq \frac{1}{2||\theta^{(n)}||}\,||\theta||^2 + \frac{1}{2}\,||\theta^{(n)}|| \quad \text{for all } \theta \in \mathbb{R}^m \setminus \{0\} \tag{2}$$

$$\sqrt{\theta} \leq \sqrt{\theta^{(n)}} + \frac{1}{2\sqrt{\theta^{(n)}}}\,(\theta - \theta^{(n)}) \quad \text{for all } \theta \in \mathbb{R}_{++} \tag{3}$$

$$v^{\mathrm{T}}\theta \leq \frac{1}{2}\left(v^{\mathrm{T}}\theta - v^{\mathrm{T}}\theta^{(n)} + 1\right)^2 + v^{\mathrm{T}}\theta^{(n)} - \frac{1}{2} \quad \text{for all } \theta \in \mathbb{R}^m \tag{4}$$

Note that $\theta^{(n)}$ is a fixed scalar/vector and should be nonzero depending on the context.

Problem 2

Section 1.4, Problem 21 of the Textbook